Sustainable Development Fund

C., GOURIEROUX (1)

September, 6, 2007
Preliminary version

---

1CREST, CEPREMAP, and University of Toronto.
This version is an incentive for open discussions. All comments are welcome. This research has benefited of the support of the chair Calyon-EDF : "Finance and Sustainable Development".
The cleaning up costs after closure of chemical or nuclear plants are large. They have to be taken into account in the production costs and in the pricing of the corporate, owner of such plants. Moreover, it is necessary to create a reserve account to cover these future large and long term risks. The aim of this paper is to explain how the management of these financing plans can be delegated to appropriate funds, called sustainable development funds (s.d.funds). These funds would be authorized to invest on risky financial assets, but would be submitted to appropriate management restrictions and to industrial and financial risk control in order to ensure that the cleaning up is completely financed at the date of closure of the plant.

We discuss the various management restrictions which can be introduced, but also the ownership transfer, when the plant is sold, the future of the fund when there is a failure of the corporate owner of the plant, or the respective roles of industrial and financial regulators.

Keywords: Fund, Sustainable Development, Regulation, Pollution.
1 Introduction

Different types of funds have been introduced on financial markets with various objectives and regulations. Examples are pension funds created to finance future pensions, mutual funds, which provide managed portfolios to individual investors, or hedge funds with more speculative objectives. These funds are submitted to more or less severe regulatory constraints. For instance, pension funds can be forced to invest only in bonds with an investment rating. Mutual funds may be invested in bonds or stocks with often specified proportions and without short sell. Hedge funds can be invested in a much larger set of assets including options, swaps, CDO's; short sell is allowed, but there is generally an announced limit for the magnitude of this leverage effect.

The aim of this paper is to introduce another type of fund built to hedge perfectly a future investment or cost. A typical example is a fund constructed to cover the future cost of cleaning up and regeneration of soils following the closure of a chemical plant, a nuclear plant or an opencast mining exploitation\(^2\). For this reason, they are called sustainable development funds (s.d.fund) in the rest of the paper\(^3\). The risk to be hedged has several characteristics, which will explain the design of the fund.

i) The cost to be hedged is often very large, which justifies a progressive financing plan.

ii) This is often a long term risk with an horizon of 10-40 years. It is difficult to constrain the fund in investing on riskfree asset only, while risky assets can provide better return in the long run without a large increase of volatility due to a possible diversification over time.

iii) It seems preferable to attach a fund to a given plant, and not to a pool of plants. First, it will facilitate the evaluation of the real production cost of this plant, which has to include future cleaning up. Second,\(^2\)The cleaning up is already mandatory for mining exploitation located in native territories, i.e. aboriginal territories in Australia, native indian or inuit territories in Canada. A typical example is the world largest bauxite mining located at Weipa in north Queensland.\(^3\)The abbreviation s.d.fund has been retained instead of sdf to insist on the portfolio interpretation of the fund, and distinguish from the organisms created to promote renewable energy, clean technologies or National Parks for instance in Pennsylvania, Hong-Kong, or Wales.

2

2

3
the plant can be sold to another firm within the period, and it will be easier to precise the becoming of the attached fund in this situation. As a consequence, we cannot expect to diversify the fund over different plants.

iv) The greater importance of sustainable development can explain why such funds may become highly recommended, or even mandatory. Their management has clearly to be controlled to avoid too large risk taken on financial markets, and likely to be submitted to the Basel regulation. They have also to be insured against a possible failure of the firm owner of the plant.

v) Finally, it is difficult to know precisely the date of the closure of a plant and the cleaning up cost twenty years in advance. A system of margin calls has to be introduced to account for the more accurate expected date of closure and expected cost when time goes on.

From a financial point of view, we have to explain how to cumulate progressively enough reserve to cover the cleaning up cost while using the investment opportunities offered by financial markets. In Section 2, we consider this question when both the date of closure and cleaning up cost are known a priori. Then, the analysis is extended in Section 3 to uncertain date of closure and cleaning up cost. The description of a complete structure including the firm owner of the plant, the special vehicle underlying the fund, the insurance company, the management of required capital in the spirit of Basel II is provided in Section 4. We also discuss some rules which might be adopted when the plant is sold to another firm, or when there is a failure of the firm owner of the plant. Section 5 concludes.

2 Fund management when the date of closure and cleaning up cost are known in advance.

In this section, we consider the limiting case in which the date of closure and cleaning up cost are known in advance. We denote by \( T_0 \) the date of creation of the fund, often equal to the date of creation of the plant \(^4\), \( T_0 + H \) the

\(^4\)See, however, the discussion in Section 4 when the plant is sold out, and the date of creation of the fund can be the purchasing date of the plant.
date of closure and $X$ the known cost. The time unit is the month for the compatibility with the Basel II practice. In the presentation, we neglect the management cost and the insurance cost to focus on financial features.

### 2.1 Financing profile

Let us first consider a fund invested in riskfree assets only, that is in $T$-bonds with maturity $T_0 + H$. We denote by $B(t, T_0 + H)$ the price at time $t$ of a zero-coupon bond paying 1 Euro at $T_0 + H$. Different financing profiles can be considered as follows:

i) **Linear financing profile**

The total cost is separated into $H$ equal costs $X/H$, each of them being covered at a given date. More precisely, at date $T_0 + h$, $h = 0, \ldots, H - 1$, the fund is asking for $X/H B(T_0 + h, T_0 + H)$ to the firm, buys $X/H$ zero-coupon bonds with maturity $T_0 + H$, which are introduced and kept in the portfolio. Thus, at time $t + h$, the portfolio includes $hX/H$ zero-coupon bonds with maturity $T_0 + H$, which are the consequences of the zero-coupon bonds introduced at the previous dates.

ii) **Geometric financing profile**

The total cost is desagregated into $H$ costs with geometric patterns $X \rho^{h-1} \frac{1 - \rho}{1 - \rho^H}$, $h = 1, \ldots, H$, $\rho > 0$. At time $T_0 + h$, the fund is asking for $X \rho^{h-1} \frac{1 - \rho}{1 - \rho^H} B(T_0 + h, T_0 + H)$ to the firm and buys $X \rho^{h-1} \frac{1 - \rho}{1 - \rho^H}$ zero-coupon bonds with maturity $T_0 + H$, kept in the portfolio. At time $T_0 + h$, the portfolio includes $X \frac{1 - \rho^h}{1 - \rho^H}$ zero-coupon bonds with maturity $T_0 + H$.

Such financing profiles can be considered as analogues of amortizing schemes encountered in accounting. The linear profile is the limiting case of geometric profile, when $\rho = 1$. When $\rho < 1$, more financing is done at the first dates.

These riskfree financing profiles will be used as benchmarks for the fund management including risky assets.
2.2 The portfolio management

Let us fix a benchmark financing profile such as a linear or geometric profile. This profile is denoted

\[ X^\ast_{T_0+h} = X^\ast(T_0 + h, \rho, X) = X \frac{1 - \rho^h}{1 - \rho^H} B(T_0 + h, T_0 + H), \]

\[ h = 0, \ldots, H - 1. \]

The fund portfolio is calibrated on this profile in the sense that the value of the portfolio invested at the beginning of period \((T_0 + h, T_0 + h + 1)\) is \(W_{T_0+h} = X^\ast_{T_0+h}\). Then, this portfolio is invested in riskfree and risky assets and can be updated under self-financing constraint between \(T_0 + h\) and \(T_0 + h + 1\). The portfolio value at \(T_0 + h + 1\) is denoted \(W^\ast_{T_0+h+1}\). Generally, it differs from the value obtained if the whole portfolio were invested in the riskfree zero-coupon bond with maturity \(T_0 + H\). It also differs from the next benchmark \(W_{T_0+h+1} = X^\ast_{T_0+h+1}\). This difference has two components, that are the financing call corresponding to a riskfree strategy plus a residual component due to the more or less efficient portfolio management in the risky assets.

The portfolio management between \((T_0 + h, T_0 + h + 1)\) has to be submitted to different restrictions, which concern:

i) the type of assets, which can be introduced in the portfolio;

ii) the short sell limitations;

iii) a condition of required capital compatible with Basel II rule.

Let us discuss more carefully this latter issue, which is generally written in terms of Value-at-Risk (VaR), that is, on a conditional quantile of the future portfolio value with an allocation crystallized at date \(T_0 + h\). At date \(T_0 + h\), the budget \(W_{T_0+h} = X^\ast_{T_0+h}\) is allocated between \(n\) risky assets and the riskfree asset for time-to-maturity one. \(^5\) These allocations are denoted \(a_{T_0+h}\) and \(a_{0,T_0+h}\), respectively. This allocation satisfies the budget constraint:

\[ a_{0,T_0+h} + a'_{T_0+h} p_{T_0+h} = X^\ast_{T_0+h}, \]

\(^5\)In the Basel II regulation, the horizon is short and in particular the zero-coupon bond with maturity \(T_0 + H\) is considered as a risky asset among others.
where $'$ denotes transpose and $p_{T_0+h}$ is the vector of prices of the different risky assets at $T_0 + h$. The future value of the crystallized portfolio is:

$$
\tilde{W}_{T_0+h+1} = a_{0,T_0+h}/B(T_0 + h, T_0 + h + 1) + a'_{T_0+h}p_{T_0+h+1}
$$

$$
= W_{T_0+h}/B(T_0 + h, T_0 + h + 1)
$$

$$
+ a'_{T_0+h}[p_{T_0+h+1} - p_{T_0+h}/B(T_0 + h, T_0 + h + 1)]
$$

$$
= W_{T_0+h}/B(T_0 + h, T_0 + h + 1) + a'_{T_0+h}\Delta \tilde{p}_{T_0+h+1},
$$

where $\Delta \tilde{p}_{T_0+h+1} = p_{T_0+h+1} - p_{T_0+h}/B(T_0 + h, T_0 + h + 1)$ denotes the net gains on risky assets. For a given risk level $\alpha$, the VaR is the amount of reserve, which has to be introduced to avoid a loss with probability $\alpha$. It is defined by:

$$
P_{T_0+h}[\tilde{W}_{T_0+h+1} + VaR_{T_0+h}(\alpha) < 0] = \alpha,
$$

where $P$ denotes the historical probability conditional on the information available at time $T_0 + h$.

In the framework of a s.d. fund, it is natural to fix a rather small risk level such as 1% and a level of VaR close to the next financing call. Then, the portfolio management will be constrained by (2.2), which will avoid too large risk taken by the portfolio manager. As usual, the fund will have to provide to the financial regulator (and the market) a sufficient information to check that the VaR constraints have been taken into account. This includes information, which has to be provided at the creation of the fund such as:

i) the selected benchmark financing profile;

ii) the selected profile for the $VaR_{T_0+h}$ (1%), if the risk level $\alpha = 1\%$ is retained by the financial regulator.

This also includes at each date $T_0 + h$ the precise portfolio allocation $a_{0,T_0+h}, a_{T_0+h}$.

The need for a small risk level is due to the myopic control retained in Basel II and to the long term risk that is considered in our framework. Loosely speaking, a 30-years risk corresponds to 360 months. Thus, at $\alpha = 1\%$, we expect that the Basel II reserve would be insufficient approximately 4-times. With $\alpha = 5\%$, the reserve will be insufficient in approximately 18-times, which is too frequent for an appropriate regulation.
2.3 An illustration

As an illustration of the fund portfolio management, let us assume a flat term structure with a zero-riskfree rate and independent net gains with a normal distribution with mean $\mu$ and volatility matrix $\Omega$ independent of the date. The Gaussian VaR is given by:

$$ VaR_{T_0+h}(\alpha) = -W_{T_0+h} - a'_{T_0+h}\mu - \Phi^{-1}(\alpha) (a'_{T_0+h}\Omega a_{T_0+h})^{1/2}, $$

(2.3)

where $\Phi$ denotes the cumulative distribution function of the standard normal.

Let us now assume a linear benchmark financial profile and the $VaR_{T_0+h}(1\%)$ fixed at the benchmark financing call. We have:

$$ W_{T_0+h} = X \frac{h}{H} \quad \text{and} \quad VaR_{T_0+h}(1\%) = X/H. $$

The regulatory constraint becomes:

$$ \frac{X}{H} = -X \frac{h}{H} - a'_{T_0+h}\mu - \Phi^{-1}(1\%)(a'_{T_0+h}\Omega a_{T_0+h})^{1/2}, $$

(2.4)

and implies a restriction on the admissible allocations $a_{T_0+h}$.

For instance, if the portfolio manager selects a mean-variance efficient allocation at horizon $1$, then,

$$ a^*_{T_0+h} = 1 A_{T_0+h}^{-1} \mu, $$

(2.5)

where $A_{T_0+h}$ is the risk aversion coefficient for period $T_0 + h$. The regulatory constraint applied to this mean-variance efficient allocation is:

$$ \frac{X \, h + 1}{H} = -a'_{T_0+h}\mu - \Phi^{-1}(1\%)(a'_{T_0+h}\Omega a_{T_0+h})^{1/2} $$

$$ = \frac{1}{A_{T_0+h}} [ -\mu' \Omega^{-1} \mu - \Phi^{-1}(1\%)(\mu' \Omega^{-1} \mu)^{1/2} ], $$

or equivalently the implied risk aversion is equal to:

$$ A^*_{T_0+h} = \frac{H(\mu' \Omega^{-1} \mu)^{1/2}[-\Phi^{-1}(1\%) - (\mu' \Omega^{-1} \mu)^{1/2}] \, X(h + 1)}{X(h + 1) \, X}, $$

(2.6)
whenever $-\Phi^{-1}(1\%) \geq \mu'\Omega^{-1}\mu$.

By fixing a constant level of VaR, whereas the size of the managed portfolio increases, the portfolio manager is forced to be more risk averse closer to the maturity (i.e. $A_{T_0+h}$ is a decreasing function of $h$). The value of the portfolio at time $T_0 + h + 1$ is:

$$W_{T_0+h+1}^* = W_{T_0+h+1} = \frac{X(h+1)}{H} + a_{T_0+h}^\mu \Delta \tilde{p}_{T_0+h+1}$$

$$= \frac{X(h+1)}{H} + \frac{X(h+1)\mu'\Omega^{-1}\Delta \tilde{p}_{T_0+h+1}}{H(\mu'\Omega^{-1}\mu)^{1/2}[-\Phi^{-1}(1\%) - (\mu'\Omega^{-1}\mu)^{1/2}]}.$$

This future value follows a Gaussian distribution with mean

$$\frac{X(h+1)}{H} + \frac{X(h+1)(\mu'\Omega^{-1}\mu)^{1/2}}{H[-\Phi^{-1}(1\%) - (\mu'\Omega^{-1}\mu)^{1/2}]}$$

and variance

$$\left[\frac{X(h+1)}{H}\right]^2 \frac{1}{[-\Phi^{-1}(1\%) - (\mu'\Omega^{-1}\mu)^{1/2}]^2}.$$ 

The mean value is strictly larger than the benchmark $\frac{X(h+1)}{H}$ corresponding to a riskfree strategy. Moreover, the expected surplus increases with $h$ and with the Sharpe performance of the market portfolio, i.e. $\mu'\Omega^{-1}\mu$.

### 2.4 The surplus and losses

As mentioned above the portfolio value at $T_0 + h + 1$, that is $W_{T_0+h+1}^*$, generally differs from both the benchmark value $W_{T_0+h+1}$ and the value $W_{T_0+h} - B(T_0 + h, T_0 + H)$ corresponding to the complete riskfree investment in zero-coupon bond with maturity $T_0 + H$. We get:
\[
W^*_T_{0+h+1} - W_{T_0+h+1} \\
= \left( W^*_T_{0+h+1} - W_{T_0+h} \frac{B(T_0 + h + 1; T_0 + H)}{B(T_0 + h, T_0 + H)} \right) \\
+ \left[ W_{T_0+h} \frac{B(T_0 + h + 1, T_0 + H)}{B(T_0 + h, T_0 + H)} - W_{T_0+h+1} \right].
\]

The first component represents the surplus or loss due to the management including risky assets, whereas the second component corresponds to the benchmark financing call and is negative.

3 Fund management with uncertain date of closure and cleaning up cost

3.1 Cost prediction

At a given date the cleaning up cost depends on the number and types of pollutants, of the pollution rate in each pollutant and the surface to be treated. It could be written as \(X_t(\theta_t)\), where \(\text{dim } \theta_t = M_t\) is the number of pollutants and \(\theta_t\) denotes the magnitude of pollution, viewed as an appropriate summary of pollution rate and surface.

The cost function is not additive:

\[X_t(\theta_t) \neq x_{0,t} + x'_t \theta_t, \text{ say,}\]

since a cleaning up treatment can concern several pollutants together. The aim of this section is not to discuss the specification of the cleaning up cost function, but to explain why the cost is difficult to predict at long horizon. The difficulties are due to the nonstationarity of the cost with the age of the plant and with time, and also to a lack of time series data on cleaning up costs.

The reasons for nonstationarity are threefold.

i) We can expect an increase of pollution with the age of the plant, and thus an increase of the cost.
ii) New cleaning up treatments can appear during the period and diminish the cost.

iii) The rules for pollution can change. For instance the list of pollutants can be modified and the threshold rates for defining pollution can be diminished.

However, the financing profile can be updated at each date. Thus, we need a reasonable sequence of prediction of cleaning up costs tending to $X_{T_0+H}(\theta_{T_0+H})$, and not necessarily unbiased predictions $T_0+hX_{T_0+H}(\theta_{T_0+H})$ of this cost performed at dates $T_0 + 1, T_0 + 2, \ldots$.

A pragmatic solution consists in replacing such an unbiased prediction $T_0+hX_{T_0+H}(\theta_{T_0+H})$ by an evaluation of the cleaning up cost if it was to be done at date $T_0 + h$, that is by $T_0+hX_{T_0+h}(\theta_{T_0+h})$. This approximation is a biased prediction of $X_{T_0+H}(\theta_{T_0+H})$, since it neglects the future technological advances, the rule modifications.... These biases will be progressively diminished and taken into account by means of the financing calls.

3.2 The financing profile

When the date of closure and cost are uncertain, we can at each date $T_0 + h$ define an approximated date of closure $T_0 + H_{T_0+H}$, say, and a benchmark cleaning up cost, $X_{T_0+h}$, say (for instance equal to $T_0+hX_{T_0+h}(\theta_{T_0+h})$). Let us now explain how to extend the linear and geometric benchmark financial profiles corresponding to riskfree investment in the zero-coupon bond with the appropriate maturity.

i) Linear financing profile

The formula is similar to the formula in Section 2.1. i), but both the cost and maturity have to be updated. At date $T_0 + h$, the fund is asking for

$$hX_{T_0+h} B(T_0 + h, T_0 + H).$$

ii) Geometric financing profile

With a fixed rate $\rho$, the fund is asking for
\[
\frac{1 - \rho^h}{1 - \rho^{H_{T_0+h}}} X_{T_0+h} B(T_0 + h, T_0 + H) = X^*(T_0 + H_{T_0+h}, \rho, X_{T_0+h}) = X^*_{T_0+h}.
\]

Due to the updating, the financing profile is not necessarily an increasing function of time. It can decrease and the financial call can become negative if the date of closure increases, or if the approximated cost decreases.

3.3 The portfolio management

As in Section 2.2, the value of the portfolio invested at the beginning of period \((T_0 + h, T_0 + h + 1)\) is fixed at the benchmark financing profile, that is, \(W_{T_0+h} = X^*_{T_0+h}\). The information to be provided to the regulator and the market is now modified. It includes

i) the type of selected benchmark profile, that is the value of \(\rho\);

ii) at each date \(T_0 + h, h = 0, 1, \ldots\), the values of the approximated maturity \(H_{T_0+h}\) and cost \(X_{T_0+h}\);

iii) the selected profile of \(VaR_{T_0+h}(1\%)\).

The \(VaR_{T_0+h}(1\%)\) can be fixed neither to the last financing call, which can be negative, nor to the future financing call, which is now unknown and could be negative. The simplest extension of the definition given in Section 2.3 fixes it to the financing call computed with crystallized maturity and cost:

\[
VaR_{T_0+h}(1\%) = X_{T_0+h} B(T_0 + h, T_0 + H),
\]

for a linear benchmark financing profile,

\[
VaR_{T_0+h}(1\%) = \frac{\rho^h - \rho^{h+1}}{1 - \rho^{H_{T_0+h}}} X_{T_0+h} B(T_0 + h, T_0 + H),
\]

for a geometric benchmark financing profile.

Since the \(VaR\) levels depend significantly on the announced maturity and cost, the proposed values have to be as objective as possible, and controllable.
by the regulator. This implies that at least the announced cost has to be fixed by a set of independent experts and that the information on the cost has also to include at each date $T_0 + h$, the list of considered pollutants, the pollution rates, the concerned surfaces... Such information should be public, since it is a required basis for building databases of cleaning up costs and their evolutions, or to check the coherency of the financing profiles of the different funds. The control of the approximated cleaning up costs by plant cannot be done by the usual financial regulators, which are specialized in financial risks, not in industrial risks. There is a need for another regulator, i.e. an industrial regulator, to control with the help of experts the proposed financing profiles. It could take the form of a "Sustainable Development Authority". Since these industrial expertises are costly and take time, they could be required at a yearly frequency, which differs from the monthly frequency retained by Basel II financial regulation.

3.4 The allocation of surplus and losses

We have noted in Section 2.4 that the portfolio value at date $T_0 + h + 1$, i.e. $W_{T_0 + h + 1}$, differs from the benchmark value $W_{T_0 + h + 1}$. In this section, we explain how the difference is reallocated between the updating of the various fund accounts and the financing and margin calls of the corporate.

i) The oversized reserve account

As previously mentioned, the regulatory reserve level is computed in a myopic perspective, and can not be sufficient to cover the financial losses appearing during the whole period. For this reason, it is important that the fund manages also an additional reserve account. Except during the last months close to maturity, this account has to ensure a reasonable level of additional reserves, between $R_{1,T_0+h}$ and $R_{2,T_0+h}$, say, with $R_{2,T_0+h} > R_{2,T_0+h}$. These levels have to be defined at the creation of the fund. For instance, for a 40-years maturity fund, they could be fixed to

$$R_{1,T_0+h} = 2VaR_{T_0+h}(1%), R_{2,T_0+h} = 4VaR_{T_0+h}(1%).$$

This oversized reserve account has another purpose; indeed, by an appropriate allocation of surplus and losses, it can be used to get total calls of corporate, generally strictly smaller than the benchmark financing call, that is to reveal that the investments in risky assets are (generally) profitable.
ii) The allocation

Three accounts are associated with the fund, that are
• the invested portfolio account;
• the oversized reserve account;
• the regulatory reserve account.

They are submitted to different investment restrictions. The regulatory reserve account receives no interest rate. The oversized reserve account could be invested in riskfree $T$-bond with short time-to-maturity 1. The fund portfolio can be invested in riskfree or risky assets. At the end of period $(T_0 + h; T_0 + h + 1)$, the states of the different accounts are:

fund portfolio : $W_{T_0+h+1};$
oversized reserve account : $R^*_{T_0+h+1} = R_{T_0+h}/B(T_0 + h, T_0 + h + 1);$
where $R_{T_0+h}$ denotes the level of oversized reserve at $T_0 + h$, with $R_{1,T_0+h} \leq R_{T_0+h} \leq R_{2,T_0+h}$.
regulatory reserve : $\text{VaR}_{T_0+h}(1%).$

At the beginning of period $(T_0 + h + 1, T_0 + h + 2)$, the state of the accounts will be:
fund portfolio : $W_{T_0+h+1};$
oversized reserve account : $R_{T_0+h+1};$
with $R_{1,T_0+h+1} \leq R_{T_0+h+1} \leq R_{2,T_0+h+1};$
regulatory reserve : $\text{VaR}_{T_0+h+1}(1%).$

From a regulatory point of view, these accounts have to be updated in a given sequence, beginning with the updating of the regulatory reserve account by means of the oversized reserve account, then the updating of the oversized reserve account and fund portfolio account by means of the invested fund portfolio and total call.

Let us now explain how to fix the future oversized reserve level and the benchmark financing and margin calls. By definition the benchmark financing call is equal to:
\[ BC_{T_0+h+1} = W_{T_0+h+1} - W_{T_0+h} \frac{B(T_0 + h + 1, T_0 + H_{T_0+h+1})}{B(T_0 + h, T_0 + H_{T_0})}. \]

The total value of the accounts at the end of period \((T_0 + h, T_0 + h + 1)\) plus the benchmark financing call is:

\[ W^*_{T_0+h+1} + R^*_{T_0+h+1} + VaR_{T_0+h}(1\%) + BC_{T_0+h+1}. \]

The quantities \(VaR_{T_0+h}(1\%)\) and \(W_{T_0+h+1}\) are fixed by regulation and contract, respectively. Thus, the residual part existing for oversized reserve and margin call is:

\[ \hat{R}_{T_0+h+1} = W^*_{T_0+h+1} + R^*_{T_0+h+1} + VaR_{T_0+h}(1\%) - BC_{T_0+h+1} - VaR_{T_0+h+1}(1\%). \]

The levels of oversized reserve and margin call (MC) depend on the location of \(\hat{R}_{T_0+h+1}\) with respect to contractual thresholds \(R_{1,T_0+h+1}\) and \(R_{2,T_0+h+1}\).

If \(\hat{R}_{T_0+h+1} > R_{2,T_0+h+1}\), we get

\[ R_{T_0+h+1} = R_{2,T_0+h+1}, \quad MC_{T_0+h+1} = -\left[ \hat{R}_{T_0+h+1} - R_{2,T_0+h+1} \right]; \]

If \(R_{1,T_0+h+1} \leq \hat{R}_{T_0+h+1} \leq R_{2,T_0+h+1}\), we have

\[ R_{T_0+h+1} = \hat{R}_{T_0+h+1}, \quad MC_{T_0+h+1} = 0. \]

If \(\hat{R}_{T_0+h+1} < R_{1,T_0+h+1}\), we have:

\[ R_{T_0+h+1} = R_{1,T_0+h+1} \quad \text{and} \quad MC_{T_0+h+1} = R_{1,T_0+h+1} - \hat{R}_{T_0+h+1}. \]

Then the total financing call will be the sum of the benchmark and margin calls.

We can expect that the first case is the more frequent. Then, the margin call will be negative and correspond to a kind of bonus with respect to the benchmark financing profile. The updating oversized reverse system is constructed in order that for instance the invested portfolio can support consecutive losses of twice the VaR without implying positive margin call. Then, this account can be refilled by means of the profit of the invested fund portfolio and so on.
4 The fund and its structural environment

The specificity of a s.d.fund compared to mutual funds or pension funds is to be both a fund invested on financial markets and a debt reimbursement. This explains why it is important to discuss in detail the questions of property transfer, or the consequence of failure of the corporate owner of the plant. This is the aim of this section, which finishes with a description of the complete structure of the fund and of the money transfers.

4.1 Shareholder

In the simplest structure, the corporate owner of the plant is the single shareholder of the fund. Special budget lines appear in its balance sheet at the end of each year $T_0 + h - 1$, say. They describe the current state of debt reimbursement, that is, the amount of debt estimated by $X_{T_0+h}B(T_0+h, T_0+H_{T_0+h})$ among Liabilities, the total value of the fund among Assets, which is decomposed into the current value of the invested fund portfolio $W_{T_0+h}$ and the residual part $X_{T_0+h}B(T_0 + h, T_0 + H_{T_0+h}) - W_{T_0+h}$.

It also includes the call, generally appearing as a liability. This part of the balance sheet is summarized in Table 1.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>invested fund portfolio</td>
<td>Debt</td>
</tr>
<tr>
<td>Residual part</td>
<td>call</td>
</tr>
</tbody>
</table>

For instance, at the creation of the fund, the balance sheet for a linear benchmark profile is the following:

Table 2 : Balance Sheet at the Date of Creation of the Fund
\begin{tabular}{|c|c|}
\hline
Asset & Liability \\
\hline
\text{IFP} : \frac{X_{T_0}}{H_{T_0}} B(T_0, T_0 + H_{T_0}) & \text{Debt} : X_{T_0} B(T_0, T_0 + H_{T_0}) \\
\text{RP} : X_{T_0} \left(1 - \frac{1}{H_{T_0}}\right) B(T_0, T_0 + H_{T_0}) & \text{Call} : 6 \frac{X_{T_0}}{H_{T_0}} B(T_0, T_0 + H_{T_0}) \\
\hline
\end{tabular}

The call include the benchmark financing call, that is \( \frac{X_{T_0}}{H_{T_0}} B(T_0, T_0 + H_{T_0}) \) plus a call for the VaR, equal to \( \frac{X_{T_0}}{H_{T_0}} B(T_0, T_0 + H_{T_0}) \) and a call for the oversized reserve account, that is \( 4 \frac{X_{T_0}}{H_{T_0}} B(T_0, T_0 + H_{T_0}) \).

The balance sheet of the fund will feature a symmetric presentation, which has to distinguish the allocation of the invested portfolio and to highlight the state of the reserve accounts.

\section*{4.2 Ownership transfer}

When the plant on which the s.d.fund is backed is sold to another corporate, two solutions can be proposed for the fund.

i) First, the shares of fund are sold with the plant to the new owner. The fund portfolio management continues under the same conditions and we have just a change of shareholder of the fund.

ii) Second, the fund is closed, its corresponding value attributed to the corporate selling the plant and new s.d.fund created by the buyer. This is only possible if the residual lifetime of the plant is sufficiently large, say larger than 10 years, to get monthly calls with reasonable magnitude. The advantage of this second solution for the buyer is to distribute the cleaning up cost on periods after the purchasing date, and thus to get a lower immediate purchasing cost. It implies a change of benchmark financing profile, since the time-to-maturity at the creation of the new fund is diminished, and the geometric rate of the financing profile can be modified.
4.3 Failure

A specificity of a s.d.fund compared to mutual funds or pension funds is to be both a fund and a debt reimbursement. Thus, it would be natural in case of failure to apply the practices for debt regulation. The failure will arise when there is a severe default of payment for one debt of the plant owner, including possibly a default on the financing calls of the fund. Then, two types of situations can be distinguished.

i) There is a decision to close the plant. In this case the value of the fund is often not sufficient to hedge the current cleaning up cost. The residual value of the debt is \( (X_{T_0+h} - X^*_T)^+ \). This is a kind of public debt and it is necessary to define its seniority level with respect to unpaid taxes, unpaid salaries and other types of debt.

ii) A procedure is engaged to solve the problem of failure by renegotiating the debts, looking for new owners... During this procedure the fund continues its financing and margin calls. The question of seniority of the debt associated with the fund is also opened in case of renegotiation, while the transfer of fund to new owners can arise if there is a change of ownership. [see Section 4.2].

As usual an insurance against failure has to be attached with each fund, and the insurance premium has to be included in the computation of the financing call. The seniority level and the transfer conditions have to be precisely defined to get a fair value of the insurance premium. For the same reason, all relevant information on the fund and its portfolio management has to be provided regularly to the insurance company. In the first situation, where the plant is closed, the insurance company has to cover the residual part of the debt. In the second situation, the insurance company can pay the calls of fund instead of the corporate and naturally become a partial shareholder of the fund, at least until the situation of the plant is clarified. Thus different shareholders can coexist, that are corporate and insurance companies. As a consequence, there can also exist a secondary market of shares of the fund at least among insurance compagnies.

Finally, note that the risk taken by the insurance company can be seen as a default risk on the corporate. Thus, it can be expected that a part

\[6\] The current cleaning up cost can sometimes be smaller than \( X^*_{T_0+h} \) due to the discounting taken into account in the benchmark financing profile.
of this risk can be transfered to other investors by means of Credit Default Swap (CDS) for instance. Among the needed CDS, some will be rather long term CDS, and we can expect that this transfer of risk by means of financial markets will enlarge the window of maturities for which some CDS will be traded.

4.4 The fund

The discussions of previous sections is summarized in Figure 1, in which the fund is presented in its structural environment, that includes the corporate shareholder, the insurance company, or the different types of industrial and financial regulators. Let us now discuss, who can create and manage a fund. As already mentioned a fund is essentially a managed financing plan. It resembles a credit whose amount is paid at maturity, not at the beginning, and the monthly payment can be reinvested. Alternatively, we can consider that it resembles an insurance contract with a claim arising with probability one and insurance premia which can be reinvested. Anyway, it seems natural to give the responsibility of the special vehicle to either banks, or insurance companies.

In practice for a given plant, different s.d.funds will be proposed to the corporate, with various financing profiles, management costs and insurance costs by banks and insurance companies. The corporate will have to select among these propositions the one, which seems the most appropriate.

5 Concluding remarks

In some industrial sectors, the cleaning up costs are high and have to be taken into account in the production cost, in the choice of appropriate technologies, in the valorisation of the firms and in their default risk, that is in their rating. In this paper, we have proposed to manage a financing plan of cleaning up cost by means of an appropriate vehicle, called sustainable development fund. Such delegate management allows to control that the cleaning up cost will be really covered. The counterpart of the control of the fund are twofold. First, this avoids a direct control of the firm itself and of its financial strategies. Second, it will likely diminish the global financing cost by the use of financial markets and by the high seniority level, which can likely be attributed to the debt w.r.t. sustainable development funds.
In practice the funds could be more or less regulated especially concerning the industrial regulation. It could be possible at the creation of the fund that the two parties, i.e. the corporate and the bank or insurance company, agree on an independent set of experts or agency to perform the expertise, and that such set or agency vary with the fund.