Higher Order Expectations, Illiquidity, and Short-term Trading

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Introduction

✓ Liquidity and asset pricing: role of private information.
✓ Persistent liquidity trading with heterogeneous information leads to multiple equilibria.
✓ HOEs on liquidity trading: joint theory of illiquidity and reliance on public and private information.
✓ Implications for asset pricing.
Persistence in liquidity:

✓ Well documented empirically (Hendershott and Seasholes (2009), Chordia and Subrahmanyam (2004)).

✓ Makes the aggregate demand for the asset depend on two “fundamentals”: a short- and a long-lived one.

✓ With limited risk-bearing capacity, it can create temporary “price pressure,” implying predictability (Hendershott and Seasholes (2009)).

✓ We uncover a novel implication of persistence.
Preview of results

With short term, heterogeneously informed investors and *persistent* liquidity trading multiple equilibria with different self-fulfilling levels of liquidity:

✓ With limited risk-bearing capacity, prices respond to liquidity trading “independently” from the arrival of news.

✓ Investors with short term horizon potentially face higher liquidation risk.

✓ However, the more aggressively investors respond to private information, the more prices reflect fundamentals, allowing to better disentangle the liquidity component of the aggregate demand, which facilitates forecasting the next period aggregate demand and the liquidation price, lowering the risk investors bear. . . .
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
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<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Reliance on public information</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Illiquidity</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Expected returns</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Expected volume of informational trading</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Price informativeness</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Return correlation at long horizons</td>
<td>–</td>
<td>–</td>
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<tr>
<td>Return correlation at short horizons</td>
<td>–</td>
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Related literature


✓ Ability of non-informational order imbalances to predict returns: Hendershott and Seasholes (2009), Coval and Stafford (2007).

Plan of the talk

✓ Model and notation.
✓ The 2-period market with heterogeneous information.
✓ Implications for reliance on public information.
✓ Implications for asset pricing.
Two assets are traded during 2 periods ($n = 1, 2$) by CARA informed investors (rational investors) and liquidity traders:

- ✓ Riskless asset with zero (net) return.
- ✓ Risky asset with final liquidation value $v \sim N(\bar{v}, \tau_v^{-1})$. 
✓ Continuum of investors in $[0, 1]$ have short term horizons: at $n$ they maximize

$$E \left[ - \exp \left\{ - \gamma^{-1} \left( (p_{n+1} - p_n) x_{in} \right) \right\} \mid s_{in}, p^n \right].$$

Short horizons can be justified on grounds of incentive reasons related to performance evaluation.

✓ At $n, i \in [0, 1]$ receives a signal

$$s_{in} = v + \epsilon_{in}, \quad \epsilon_{in} \sim N(0, \tau_{\epsilon}^{-1}),$$

with $\epsilon_{in} \perp \epsilon_{jn}, \epsilon_{in} \perp \epsilon_{in+1}$, and also orthogonal to all the other random variables in the model.
✓ Each investor $i$ observes the sequence of prices up to period $n$:

$$p^n \equiv \{p_t\}_{t=0}^n, \ n = 1, 2.$$  

✓ Submits a demand schedule

$$X_n(s_{in}, p_{n-1}^n, p_n) = a_n s_{in} - \varphi_n(p^n),$$

denoting the desired position in the risky asset.

✓ Convention:

$$\int_{0}^{1} s_{in} \, di = v, \text{ a. s.}$$
Liquidity traders

Liquidity traders’ demand:


✓ Liquidity trading follows an AR(1) process:

\[ \theta_n = \beta \theta_{n-1} + u_n, \]

with \( \beta \in [0, 1] \), and \( u_n \sim N(0, \tau_u^{-1}) \) orthogonal to all other random variables.

✓ Note:

\[ \beta = \begin{cases} 
1 & \theta_n \text{ follows a random walk} \\
0 & \text{Noise trading is i.i.d. across periods.} \\
\end{cases} \]

\[ u_n = \theta_n - \theta_{n-1} \]


Suppose $u_n \sim N(0, 1)$ and $\beta = 0$
Liquidity traders (ctd.)

Suppose $u_n \sim N(0, 1)$ and $\beta = 1/2$
Suppose $u_n \sim N(0, 1)$ and $\beta = 1$
Timeline

Let \( x_n \equiv \int_0^1 x_i \, di \)

<table>
<thead>
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<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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Notation

✓ Investor $i$'s expectation about the liquidation value at time $n$ is

$$E_{in}[v] \equiv E[v|s_{in}, p^n].$$

✓ Investor $i$'s forecast precision at time $n$ is

$$\tau_{in} \equiv \frac{1}{\text{Var}[v|s_{in}, p^n]}.$$

✓ The expectation based on public information only is

$$E_n[v] \equiv E[v|p^n].$$

✓ Public precision is

$$\tau_n \equiv \frac{1}{\text{Var}[v|p^n]}.$$

✓ The average expectation (consensus opinion) is

$$\bar{E}_n[v] \equiv \int_0^1 E_{in}[v] di = \alpha_{E_n} v + (1 - \alpha_{E_n}) E_n[v],$$

$\alpha_{E_n}$: optimal statistical weight to private info.
A static market
Suppose we “removed” the second period

1

- Liquidity traders submit $\theta_1 = u_1$.
- 1st period informed investors submit $x_1$.
- Market clears: $x_1 + \theta_1 = 0$.

2

- Liquidity traders submit $\theta_2 = u_2 + \beta \theta_1$.
- 2nd period informed investors submit $x_2$.
- 1st period investors revert.
- Market clears: $\Delta x_2 + \Delta \theta_2 = 0$.

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- The asset is liquidated.
A static market
Suppose we “removed” the second period

1. Static strategies, “buy and hold”:

\[ X_1(s_{i1}, p_1) = \gamma \frac{E_{i1}[v] - p_1}{\text{Var}_{i1}[v]} \]

2. Static price akin to “consensus”:

\[ p_1 = \bar{E}_1[v] + \frac{\text{Var}_{i1}[v]}{\gamma} \theta_1 = E_1[v] + \frac{\text{Var}_{i1}[v]}{\gamma} E_1[\theta_1], \text{ where } \lambda_1 = \frac{\alpha E_1}{a_1} + (1 - \alpha E_1) \frac{a_1 \tau u}{\tau_1} \]

but also, \( E_1[\theta_1] \) moves \( p_1 \) away from the efficient price (Hendershott and Seasholes (2004)).

3. The two components of the price comove positively:

\[ \text{Cov}[E_1[v], E_1[\theta_1]] > 0 \Rightarrow \text{Var}[p_1] > \text{Var}[E_1[v]] + \Lambda_1^2 \text{Var}[E_1[\theta_1]]. \]
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\[ X_1(s_{i1}, p_1) = \gamma \frac{E_i[v] - p_1}{\text{Var}_{i1}[v]} . \]

2. Static price akin to “consensus”:

\[ p_1 = \alpha E_1 \left(v + \frac{\theta_1}{a_1}\right) + (1 - \alpha E_1) E_1[v] \]

\[ = E_1[v] + \frac{\text{Var}_{i1}[v]}{\gamma} E_1[\theta_1] , \text{ where } \lambda_1 = \frac{\alpha E_1}{a_1} + (1 - \alpha E_1) \frac{a_1 \tau_u}{\tau_1} , \]

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A 2-period market
Equilibrium

Proposition

At a linear equilibrium of the market the price is given by

\[ p_n = \alpha p_n \left( v + \frac{\theta_n}{a_n} \right) + \left( 1 - \alpha p_n \right) E_n[v] \]
\[ = E_n[v] + \Lambda_n E_n[\theta_n] \]

where,

\[ \Lambda_1 = \frac{\text{Var}_i[p_2]}{\gamma} + \beta \Lambda_2, \]

while \( \Lambda_2 = 1/(\gamma \tau_{i2}) \).

Short-term investment horizons imply:

✓ A change in the inventory component of illiquidity \( \Lambda_1 \).
✓ A change in the weight assigned by the price to noisy private information \( \alpha p_1 \).
The change in $\Lambda_1$

The impact of liquidity trading persistence

In the static market:

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Then:

(a) Investors hold the asset until date 3.
(b) The price reflects the (risk-tolerance weighted) uncertainty about $v$:

$$p_1 = E_1[v] + \frac{\text{Var}_i_1[v]}{\gamma} E_1[\theta_1].$$
The change in $\Lambda_1$ (ctd.)

The impact of liquidity trading persistence

In the dynamic market, due to short horizons there is re-trading at date 2:

$$X_1(s_{i1}, p_1) = \gamma \frac{E_{i1}[p_2] - p_1}{\text{Var}_{i1}[p_2]}.$$  

(a) First period investors face uncertainty over the liquidation price which bites more, the higher is their risk aversion:

$$\frac{\text{Var}_{i1}[p_2]}{\gamma}.$$  

(b) Because of persistence in liquidity, $\theta_1$ also affects the second period investors ($\beta \theta_1$), and this “adds” to first period investors’ uncertainty over the liquidation price:

$$\beta \Lambda_2.$$  

Overall, the first period price reaction to the (expected) liquidity shock is

$$\Lambda_1 = \frac{\text{Var}_{i1}[p_2]}{\gamma} + \beta \Lambda_2.$$
The change in $\alpha P_1$

An alternative expression for the price

$$p_1 = \bar{E}_1[v] + \frac{\alpha P_1 - \alpha E_1}{a_1} E_1[\theta_1] + \frac{\alpha E_1}{a_1} \theta_1.$$

✓ Short-term trading yields a systematic departure of the price from consensus (the “static” price).

✓ What does this imply?

Corollary

At a linear equilibrium

$$\text{Cov}[p_1, v] > \text{Cov}[\bar{E}_1[v], v] \iff \alpha P_1 > \alpha E_1.$$

With short term speculation the price is either systematically drawn closer or farther away from fundamentals compared to consensus.
The change in $\alpha P_1$ (ctd.)

Strategies

Return predictability:

$$E_1[p_2 - p_1] = -\frac{\text{Var}_{i1}[p_2]}{\gamma} E_1[\theta_1] \Leftrightarrow E_1[\theta_1] = -\frac{\gamma}{\text{Var}_{i1}[p_2]} E_1[p_2 - p_1],$$

so that for any estimate of $\theta_1$, the market anticipates reversion. However,

Corollary

*At a linear equilibrium, a rational investor's strategy in the first period is given by*

$$X_1(s_{i1}, p_1) = \frac{a_1}{\alpha E_1} (E_{i1}[v] - p_1) + \frac{\alpha P_1 - \alpha E_1}{\alpha E_1} E_1[\theta_1].$$

Short term strategies depend on whether \( \{ \alpha P_1 > \alpha E_1 \implies \text{Trend chasing} \) \( \alpha P_1 < \alpha E_1 \implies \text{Contrarian} \)
Trend chasers and contrarians

Suppose $E_1[\theta_1] > 0$:

$$E_1[\theta_1] = a_1(v - E_1[v]) + \theta_1 \begin{cases} \text{Fundamentals information:} & v > E_1[v] \\ \text{Liquidity trading:} & \theta_1 > 0. \end{cases}$$

If $\alpha_{P_1} > \alpha_{E_1}$,

(a) The price reflects better the fundamentals than the static price.

(b) But the departure from the static price is due to short-term trading.

(c) Hence, short term trading is tying the price closer to fundamentals (compared to consensus) and $E_1[\theta_1] > 0$ is possibly signalling fundamentals information!

Short term investors “chase the trend.”
Multiple equilibria

Proposition

Linear equilibria always exist. If

1. $\beta \in (0, 1]$:
   
   (a) There are two equilibria in which $a_2 = \gamma \tau \epsilon$, and
   
   $a_1 \begin{cases} 
   0 < a_1^* < \gamma \tau \epsilon & \alpha_{P_1} < \alpha_{E_1} & \lambda_2 > 0 \\
   \gamma \tau \epsilon < a_1^{**} & \alpha_{P_1} > \alpha_{E_1} & \lambda_2 < 0
   \end{cases}$

   (b) Furthermore, $|\lambda_2(a_1^{**})| < \lambda_2(a_1^*)$, and prices are more informative along the low illiquidity equilibrium.

2. $\beta = 0$: only the high illiquidity equilibrium survives: $a_2 = \gamma \tau \epsilon, a_1 < \gamma \tau \epsilon$, and investors are contrarians: $\alpha_{P_1} < \alpha_{E_1}$.
The role of persistence

Intuition
An investor at 1 uses his private signal to anticipate \( p_2 \):

✓ Due to persistence:

1. Can anticipate part of the 2nd period liquidity demand:

\[
E_{i1}[p_2] = \Lambda_2 a_2 E_{i1}[v] + (1 - \Lambda_2 a_2) E_{i1}[E_2[v]] + \Lambda_2 \beta E_{i1}[\theta_1].
\]

2. Potentially face an increase in the liquidation price risk:

\[
\text{Var}_{i1}[p_2] = \text{Var}_{i1}[E_2[v]] + \Lambda_2^2 \text{Var}_{i1}[E_2[\theta_2]] + 2\Lambda_2 \text{Cov}_{i1}[E_2[v], E_2[\theta_2]].
\]

✓ However, this risk is endogenous and as \( a_1 \) increases

1. \( E_1[v] \) and (for \( a_1 \) large enough) \( E_2[v] \) are closer to the fundamentals.
2. Investors can better separate the impact of \( v \) from that of \( \theta_1 \):

\[
\frac{\partial \text{Var}[E_{i1}[\theta_1]]}{\partial a_1} < 0.
\]

3. For \( \beta > 0 \) this improves the forecast of \( E_2[\theta_2] \), reducing the uncertainty over \( p_2 \) (\( \text{Cov}_{i1}[E_2[v], E_2[\theta_2]] \) becomes negative).

✓ These two effects generate a double feedback loop yielding multiple equilibria.
Consider a 3-period model. In period 3:
\[
\int_{0}^{1} X_3(s_{i3}, p^3) di + \theta_3 = 0, \quad \text{due to normality and CARA} \Rightarrow \gamma \frac{\bar{E}_3[v] - p_3}{\text{Var}_{i3}[v]} + \theta_3 = 0.
\]
Hence,
\[
p_3 = \bar{E}_3[v] + \Lambda_3 \theta_3, \quad \text{where } \Lambda_3 = \frac{\text{Var}_{i3}[v]}{\gamma}.
\]
In period 2
\[
p_2 = \bar{E}_2[p_3] + \frac{\text{Var}_{i2}[p_3]}{\gamma} \theta_2.
\]
Substituting the above obtained expression for \( p_3 \):
\[
p_2 = \bar{E}_2 \left[ \bar{E}_3[v] + \frac{\text{Var}_{i3}[v]}{\gamma} \theta_3 \right] + \frac{\text{Var}_{i2}[p_3]}{\gamma} \theta_2
\]
\[
= \bar{E}_2 \left[ \bar{E}_3[v] \right] + \frac{\text{Var}_{i3}[v]}{\gamma} \beta \bar{E}_2 \left[ \theta_2 \right] + \frac{\text{Var}_{i2}[p_3]}{\gamma} \theta_2.
\]
Consider a 3-period model. In period 3:

\[
\int_0^1 X_3(s_i, p^3) di + \theta_3 = 0, \text{ due to normality and CARA} \Rightarrow \gamma \frac{\bar{E}_3[v] - p_3}{\text{Var}_i3[v]} + \theta_3 = 0.
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Hence,

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p_3 = \bar{E}_3[v] + \Lambda_3 \theta_3, \text{ where } \Lambda_3 = \frac{\text{Var}_i3[v]}{\gamma}.
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\[
= \bar{E}_2 \left[ \bar{E}_3[v] \right] + \frac{\text{Var}_i3[v]}{\gamma} \beta \bar{E}_2 \left[ \theta_2 \right] + \frac{\text{Var}_i2[p_3]}{\gamma} \theta_2.
\]

HOEs and reliance on public information
In period 1:

\[ p_1 = \bar{E}_1 \left[ \bar{E}_2 \left[ \bar{E}_3 [v] \right] \right] + \frac{\text{Var}_i[3][v]}{\gamma} \beta \bar{E}_1 \left[ \bar{E}_2[\theta_2] \right] + \frac{\text{Var}_i[2][p_3]}{\gamma} \beta \bar{E}_1[\theta_1] + \frac{\text{Var}_i[1][p_2]}{\gamma} \theta_1, \]

The “Beauty contest:” when \( \beta > 0 \) the equilibrium price reflects HOEs

1. Liquidation value (as in Allen, Morris, and Shin (2006)).
2. Liquidity trades in periods 1, and 2.
In period 1:

\[ p_1 = \bar{E}_1 \left[ \bar{E}_2 \left[ \bar{E}_3 \left[ v \right] \right] \right] + \frac{\text{Var}_3[v]}{\gamma} \beta \bar{E}_1 \left[ \bar{E}_2[\theta_2] \right] + \frac{\text{Var}_2[p_3]}{\gamma} \beta \bar{E}_1[\theta_1] + \frac{\text{Var}_1[p_2]}{\gamma} \theta_1, \]

The “Beauty contest:” when \( \beta > 0 \) the equilibrium price reflects HOEs

1. Liquidation value (as in Allen, Morris, and Shin (2006)).
2. Liquidity trades in periods 1, and 2.
When $\beta = 0$, only HOEs about fundamentals matter and:

$$\text{Cov}[p_1, v] < \text{Cov}[\bar{E}_1[v], v],$$

and $p_1$ relies heavily on public information (compared to consensus), as in Allen et al. (2006). However,

**Corollary**

*When $\beta > 0$, and*

$$a_1 = \begin{cases} 
  a_1^* & \text{Cov}[p_1, v] < \text{Cov}[\bar{E}_1[v], v] \\
  a_1^{**} & \text{Cov}[p_1, v] > \text{Cov}[\bar{E}_1[v], v].
\end{cases}$$

*When $\beta > 0$, HOEs about liquidity trading affect prices, and short term speculation can make prices better predictors of fundamentals (compared to consensus).*
Asset pricing implications
Momentum and reversal

Consistent with empirical evidence on return regularities:

Corollary

When $N = 2$:

1. For all $\beta \in [0, 1]$, $\text{Cov}[p_2 - p_1, p_1 - \bar{v}] < 0$.

2. For $\beta \in (0, 1)$, $\text{Cov}[v - p_2, p_1 - \bar{v}] < 0$. For $\beta = 0$, $\text{Cov}[v - p_2, p_1 - \bar{v}] = 0$.

3. For $\beta \in (0, 1)$, along the equilibrium with low illiquidity
   $\text{Cov}[v - p_2, p_2 - p_1] > 0$. Along the equilibrium with high illiquidity, for $\tau_v < \hat{\tau}_v$, there exists a value $\hat{\beta}$ such that for all $\beta > \hat{\beta}$,
   $\text{Cov}[v - p_2, p_2 - p_1] > 0$. If $\beta = 0$, $\text{Cov}[v - p_2, p_2 - p_1] < 0$. 
Asset pricing implications

Momentum and reversal: mean price path

Differently from behavioral finance literature:
Asset pricing implications
Volume and return predictability

Volume of informational trading:

\[
V_2 \equiv \int_0^1 E \left[ \left| X_2(s_{i2}, z^2) - X_1(s_{i1}, z_1) \right| \right] \, di - \int_0^1 E \left[ \left| X_2(\theta_2) - X_1(\theta_1) \right| \right] \, di
\]

Corollary

When \( N = 2, \) for all \( \beta \in (0, 1] \)

1. **The expected volume of informational trading is higher along the low illiquidity equilibrium.** When \( \beta = 0 \) only the equilibrium with a low volume of informational trading survives.

2. **Expected returns are higher along the high illiquidity equilibrium.**
A high volume of informational trading predicts momentum, consistently with Llorente et al. (2002). However,

- With high persistence ($\beta$ high $\sim$ very different trading frequencies), momentum is compatible with both trend chasing and contrarian behavior.

  $\Rightarrow$ Liquidity-motivated trades also may generate positively autocorrelated returns.

- With low persistence momentum is only compatible with trend chasing behavior.

- If investors chase the trend, they are on the equilibrium with momentum at short horizons.
Asset pricing implications
Illiquidity and expected returns

Due to the law of iterated expectations:

\[ E[p_n - p_{n-1}] = \Lambda_n E[\theta_n] - \Lambda_{n-1} E[\theta_{n-1}] . \]

This implies

Corollary

When \( u_n \sim N(\bar{u}, \tau_u^{-1}) \),

\[ E[p_1 - \bar{v}] = \Lambda_1 \bar{u} \]

\[ E[p_2 - p_1] = (\Lambda_2 (1 + \beta) - \Lambda_1) \bar{u} \]

\[ E[v - p_2] = -\Lambda_2 (1 + \beta) \bar{u} . \]
Asset pricing implications
Illiquidity and expected returns (ctd)

Denoting by $\Lambda_n^H$ and $\Lambda_n^L$, the inventory component of market illiquidity associated with the high and low illiquidity equilibrium at time $n$:

**Corollary**

$\Lambda_n^L < \Lambda_n^H$, for $n = 1, 2$.

As investors anticipate worse market conditions along the high illiquidity equilibrium,

✓ The expected compensation required to absorb the demand shock from liquidity traders in the first and second period is higher, implying a higher risk premium for first and third period returns.

✓ Numerical simulations show that a similar result also holds in the second period.
Asset pricing implications

Illiquidity and expected returns (ctd)

Figure 7: In panel (a), (b), and (c) we plot the risk premium associated with first, second, and third period returns along the high and low illiquidity equilibrium (respectively, dotted and continuous line) for $\beta \in [0, 1]$. Parameters' values are as follows: $\tau_v = \tau_u = \tau_\pi = \bar{u} = 1$, and $\gamma = 1/2$.

It is interesting to relate our results with Easley and O'Hara (2004) who show that private information induces a risk that is priced in equilibrium. In their setup, stocks for which the amount of private information is higher (compared to public signals) command a premium that serves to compensate uninformed investors for the losses they expect to make vis-à-vis informed investors because of adverse selection. A similar effect is also at work in our model. Indeed, although the illiquidity premium is only related to the inventory risk component of illiquidity $\Lambda_n$ (and thus the adverse selection component of illiquidity does not appear in (37)), as we have argued in Section 4, along the high
Further implication of the figure:

✓ As $\beta$ increases from 0 to 1, the risk premia at all trading dates increase, along each equilibrium.

✓ As $\beta$ becomes larger, a higher fraction of liquidity traders’ positions at any date remains in the market, augmenting the inventory risk born by rational investors. As a consequence, the expected premium required to absorb liquidity trades increases.
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<td>Illiquidity</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Expected returns</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Expected volume of informational trading</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Price informativeness</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Return correlation at long horizons</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Return correlation at short horizons</td>
<td>–</td>
<td>±</td>
</tr>
</tbody>
</table>
When a market is populated by short term investors,

✓ With persistence in liquidity trading and heterogeneous information, short term horizons yield multiple equilibria which can be ranked in terms of illiquidity.

✓ Implications for reliance on public and private information, and price informativeness.

✓ Implications for asset pricing: momentum, volume and return predictability, and expected returns.