Socially efficient discounting under ambiguity aversion

Johannes Gierlinger
Christian Gollier
Introduction

- Which discount rate should be used for the distant future?

- Applications: Nuclear wastes, global warming…

- “There must be something wrong with discounting”: 1,000,000 € in 200 years discounted at 8% is valued 20 cents today.

- Copenhagen Consensus versus Stern Review.

- Two problems:
  - the level of the discount rate;
  - its constancy with respect to time horizon.

- A standard consumption-based model of the yield curve to determine its level and its shape.
The three determinants of the discount rate

- Ethical dimension: intergenerational Pareto weights in the SWF.

- Preference for consumption smoothing over time + positive growth of GDP per capita (+): the marginal utility of 1 unit of consumption next period is smaller than the marginal utility of 1 unit of consumption now.

- Prudence + uncertain growth (-)

Ramsey rule: \[ r_t = \delta + \gamma \mu - 0.5 \gamma^2 \sigma^2 \]
Ambiguous growth and ambiguity aversion

- Who can estimate the $\mu$ and $\sigma^2$ for the next 200 years?
- People are ambiguity-averse.
- This paper: what is the socially efficient level and term structure of the discount rates when the growth process is uncertain?
- Conjecture: ambiguity aversion should reduce the discount rate.
On the socially efficient discount rate:
- Weitzman (1999, 2007)

On ambiguity aversion:
- Gilboa and Schmeidler (1987)
- Klibanoff, Marinacci and Mukerji (2005)
The model

- The economic growth is governed by an unknown parameter $\theta$ which can take value $1, 2, \ldots, n$ with probability $q_1, \ldots, q_n$.

- Conditional to $\theta$, the distribution of GDP/cap at date $t$ is $c_{t\theta}$.

- An investment yields $e^{rt}$ units of consumption at date $t$ per unit invested at 0.

- $\alpha^* = 0$ must be an equilibrium.

$$
\alpha^* \in \arg \max_{\alpha} \ u(c_0 - \alpha) + e^{-\delta t} V_t(\alpha)
$$
The model

- Risk aversion ($u$ concave) and ambiguity neutral:
  \[ V_t(\alpha) = \sum_{\theta=1}^{n} q_\theta Eu(\bar{c}_{t\theta} + \alpha e^{r_{it}}) \]

- RA + Ambiguity aversion ($\phi$ concave):
  \[ \phi(V_t(\alpha)) = \sum_{\theta=1}^{n} q_\theta \phi(Eu(\bar{c}_{t\theta} + \alpha e^{r_{it}})) \]

- The equilibrium condition is written as:
  \[ r_t = \delta - \frac{1}{t} \ln \left[ \frac{\sum_{\theta=1}^{n} q_\theta \phi'(Eu(\bar{c}_{t\theta}))Eu'(\bar{c}_{t\theta})}{u'(c_0)\phi'(V_t(0))} \right] \]
An analytical solution

- Specification:
  \[
  \ln c_t \sim N(\ln c_0 + \theta t, \sigma^2 t) \\
  \theta \sim N(\mu, \sigma^2_0) \\
  u(c) = c^{1-\gamma} / (1 - \gamma) \\
  \phi(V) = V^{1-\eta} / (1 - \eta)
  \]

- Solution:
  \[
  r_t = \delta + \gamma \mu - 0.5 \gamma^2 (\sigma^2 + \sigma^2_0 t) - 0.5 \eta \left| 1 - \gamma^2 \right| \sigma^2_0 t
  \]
The case of risk neutrality

- Proposition: Suppose that the agent is risk-neutral (u linear). Then,
  - Decreasing ambiguity aversion => discount rate reduced;
  - Constant ambiguity aversion => discount rate unchanged;
  - Increasing ambiguity aversion => discount rate increased.
AA = More patience + More pessimism

\[ r_t = \delta - \frac{1}{t} \ln \left[ \sum_{\theta=1}^{n} q_{\theta} \phi'(\text{Eu}(c_{t\theta})) \frac{\text{Eu}'(c_{t\theta})}{u'(c_0)\phi'(V_t(0))} \right] \]

\[ r_t = \left[ \delta - \frac{1}{t} \ln a \right] - \frac{1}{t} \left[ \sum_{\theta=1}^{n} \tilde{q}_{\theta} \frac{\text{Eu}'(c_{t\theta})}{u'(c_0)} \right] \]

\[ a = \sum_{\theta=1}^{n} q_{\theta} \frac{\phi'(\text{Eu}(c_{t\theta}))}{\phi'(V_t(0))} \]

\[ \tilde{q}_{\theta} = q_{\theta} \frac{\phi'(\text{Eu}(c_{t\theta}))}{\sum_{\tau=1}^{n} q_{\tau} \phi'(\text{Eu}'(c_{t\tau}))} \]

\[ \tilde{q} \sim_{\text{MLR}} q \text{ under AA} \]

\[ a > 1 \text{ if DAAA} \]
Assume DAAA

Then, it remains to see whether the pessimistic deterioration in the distribution of growth reduces the discount rate.

In this talk, we focus on two special cases where this is true.

In the paper, we also consider a third case based on Jewitt's notion of stochastic dominance.
Result with FSD

- Suppose that \( c_{t_1} \prec_{FSD} c_{t_2} \prec_{FSD} \cdots \prec_{FSD} c_{t_n} \).

- Then, it can be shown that the implicit distribution of \( c_t \) is FSD-deteriorated by more ambiguity aversion.

- Then, by risk aversion, the discount rate is reduced.
Result with SSD

Suppose that \( c_{t_1} \prec_{SSD} c_{t_2} \prec_{SSD} \cdots \prec_{SSD} c_{t_n} \).

Then, it can be shown that the implicit distribution of \( c_t \) is SSD-detreriorated by more ambiguity aversion.

Then, under prudence, the discount rate is reduced.
Conclusion

- The growth process is ambiguous by nature.
- Human beings are ambiguity-averse by nature.
- These two ingredients impose us to do more for the future.
- Reduce the discount rate!
- Make the term structure decreasing!