Optimal mix between pay-as-you-go and funded pension systems: the case of Luxembourg

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ABSTRACT

Keywords:

1. INTRODUCTION

2. THE PENSION SYSTEM IN LUXEMBOURG

3. ANALYSIS OF THE SALARY TRAJECTORIES

3.1 A statistical method based on clustering

π_j

Y N $Y_i = y_i$ y_i y_{iT}

i T

$P\ Y_i$ Y_i Y_i $P\ Y_i$

r

$P\ t = \beta + \beta t + \beta t + \beta t + \beta t$

$P^j\ Y_i$ i

j π_j j

π_j j

$\Omega = \beta^j \quad \beta^j \quad \beta^j \quad \beta^j \quad \beta^j \quad \pi_j \quad j = \quad r$

Y_i r

Y_i y_{it}

T L

$$L = \frac{}{\sigma}\prod_{i=}^{N}\sum_{j=}^{r}\pi_j\prod_{t=}^{T}\phi\left(\frac{y_{it}-\beta^j x_{it}}{\sigma}\right)$$

ϕ

π_j

$$\pi_j = \frac{e^{\theta_j}}{\sum_{j=}^{r} e^{\theta_j}}$$

θ_j π_j *r-1*

θ_j $\theta_1 = 0.$

$$L = \frac{}{\sigma}\prod_{i=}^{N}\sum_{j=}^{r}\frac{e^{\theta_j}}{\sum_{j=}^{r} e^{\theta_j}}\prod_{t=}^{T}\phi\left(\frac{y_{it}-\beta^j x_{it}}{\sigma}\right)$$

$= \quad L - \quad k \quad N$

k

$P \; j \; Y_i$ i j

$$P \; j \; Y_i = \frac{P \; Y_i \; j \; \pi_j}{\sum_{j=}^{r} P \; Y_i \; j \; \pi_j}$$

Y_i

$P(Y_i / j)$

3.2 The IGSS database

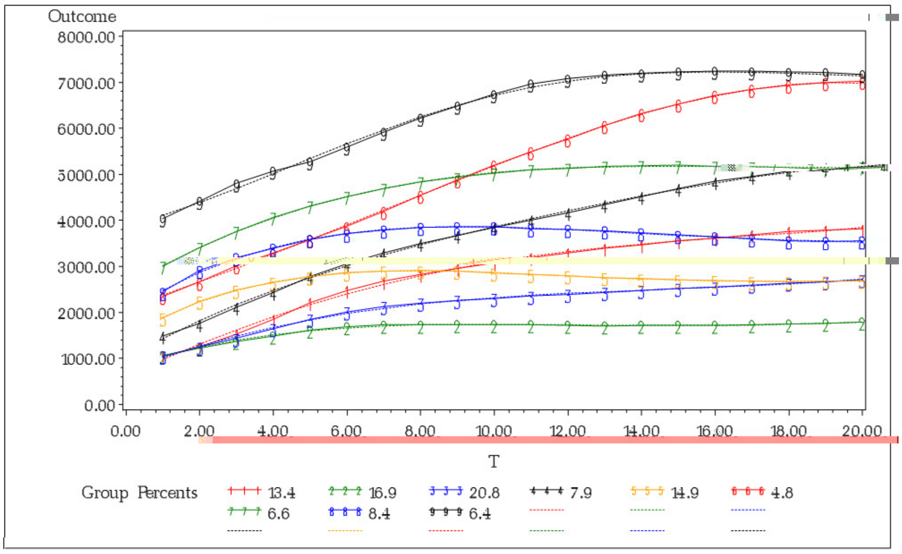


FIG 1: AVERAGE SALARY CURVES ACROSS THE 9 GROUPS

4. THE PENSION MODEL

4.1 Using the salary trajectories

$p_1 = 13.4\,\%$	$p_2 = 16.9\,\%$	$p_3 = 20.8\,\%$	$p_4 = 7.9\,\%$	$p_5 = 14.9\,\%$
$p_6 = 4.8\,\%$	$p_7 = 6.6\,\%$	$p_8 = 8.4\,\%$	$p_9 = 6.4\,\%$	

TABLE 1: GROUP SIZES

λ_i $i =$ λ_i

$y_i = P_i\ t$ β_i t $P_i\ t$

$\lambda_i =$ β_i $-$

Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
$\lambda_i = 3.07\%$	$\lambda_2 = 0.96\%$	$\lambda_3 = 1.45\%$	$\lambda_4 = 2.82\%$	$\lambda_5 = 0.19\%$
Curve 6	Curve 7	Curve 8	Curve 9	
$\lambda_6 = 2.58\%$	$\lambda_7 = 1.28\%$	$\lambda_8 = 0.48\%$	$\lambda_9 = 1.09\%$	

TABLE 2: WAGE GROWTHS IN THE NINE GROUPS

p_i

λ_i p_i'

$p_i = p_i'$ r

d *individuals*
T S

$T = 40$ $S = 20$ d

t n_i i t

$(i, \ln(n_i))$ $\ln(n_i) = a + bi.$

$$N = \quad a$$

$$d = \quad -b \quad -$$

n n_{T+S} N N_{T+S}

by

$$N_i = \frac{N}{+d^{\ i}}$$

d

BOX 1. EVOLUTION OF THE DEMOGRAPHIC INTERGENERATIONAL RATE

d

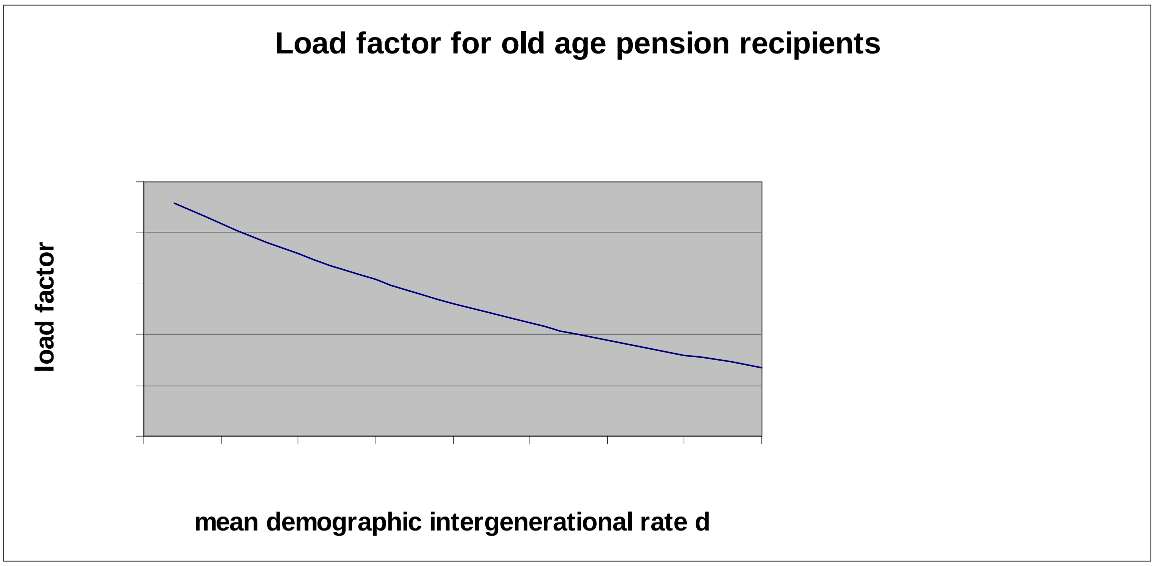


FIG. 2: LOAD FACTOR FOR OLD AGE PENSION RECIPIENTS

d *d*
d
d

4.2 Analysis of the pay-as-you-go system

τ_1 τ_1

4.2.1 Definition of the sustainability coefficient

τ_1

t

τ_1 τ_1

t

4.2.2 Evaluation of τ_1 as a function of *d*

d

S_i *i*

P_j *j* *T*

$$N_a = N\,S \;+\quad + \frac{N\,S_T}{+\,d^{\;T}}$$

k *T*

$$N_p = \frac{kN}{+\,d^{\;T+}} P_{T+} \;+\quad + \frac{kN}{+\,d^{\;T+S}} P_{T+S}$$

k

k

τ_1 *d*

$$\tau = \frac{N_p}{N_a} = \frac{\frac{k}{+d^{\;T+}} P_{T+} \;+\quad + \frac{k}{+d^{\;T+S}} P_{T+S}}{S \;+\quad + \frac{S_T}{+d^{\;T}}}$$

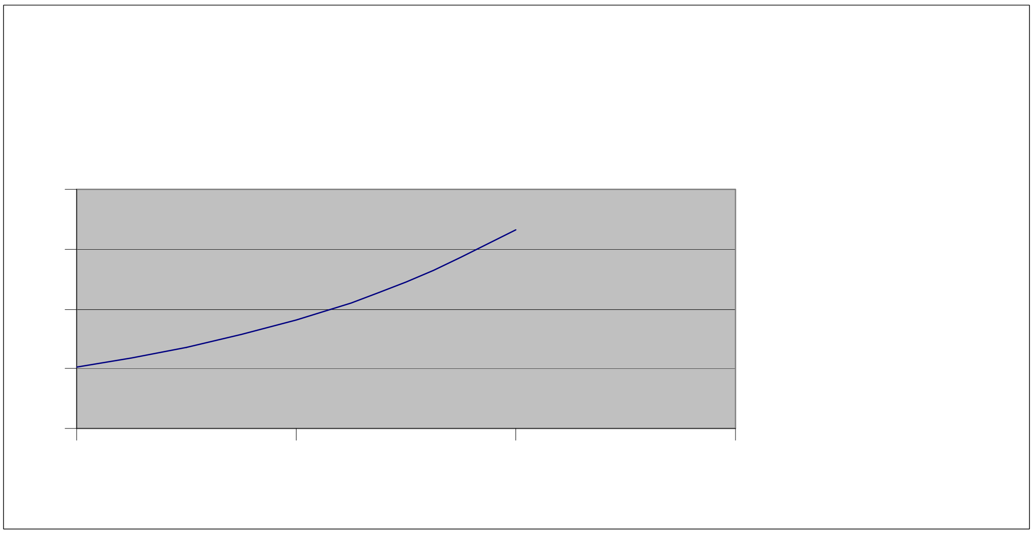


FIG 3: SUSTAINABILITY COEFFICIENT AS A FUNCTION OF THE DEMOGRAPHIC RATE

$$E\;\tau \quad = \qquad\qquad \sigma\;\tau \quad =$$

$CV\ \tau \quad = \text{——} =$

τ_1

d τ_1

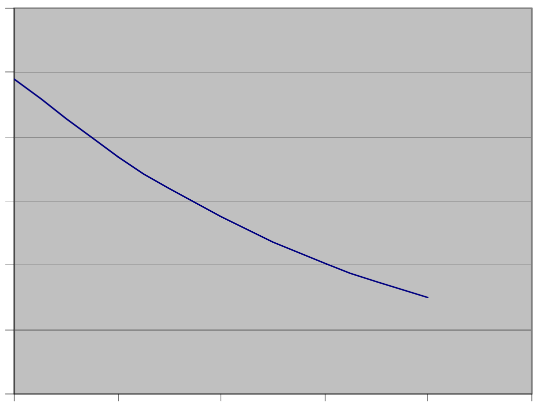


FIG 4: RATE OF PENSION CONTRIBUTIONS AS A FUNCTION OF d

k

c_1

$c \ =$ γ

$$P = \gamma P \qquad \frac{P}{S} = c$$

$$\frac{\gamma P}{S} = c \ \ <=> \gamma c \ = c \ \ <=> \gamma = \frac{c}{c}$$

k d c_0 γ

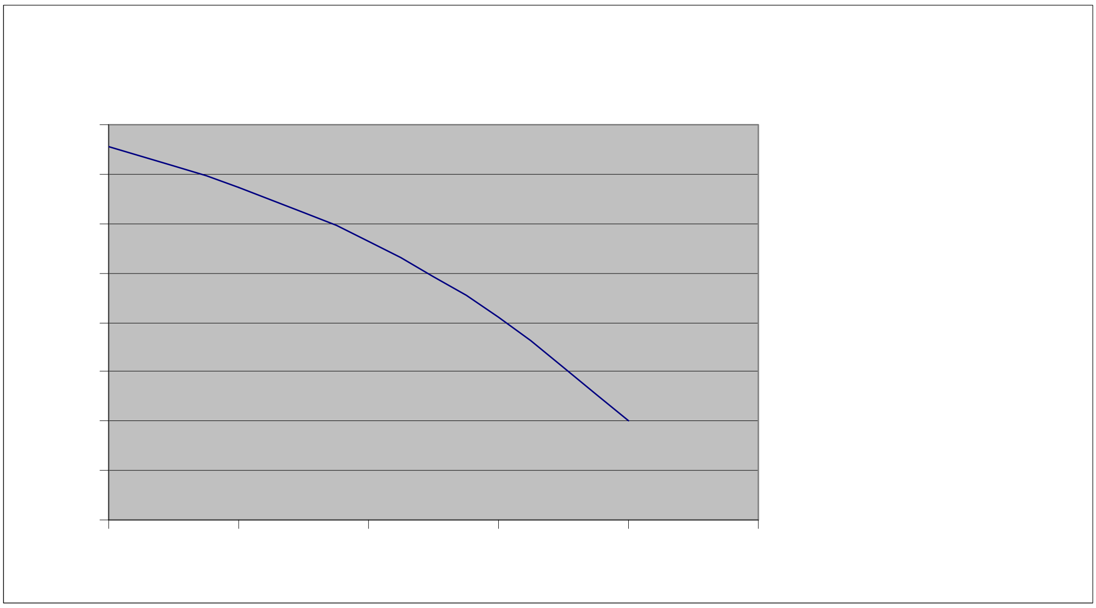


FIG 5: FALL OF THE REPLACEMENT RATE AS A FUNCTION OF d

5. A LESS RISKY APPROACH: MIXTURE OF PAY-AS-YOU-GO AND CAPITALIZATION

x

(1-x)

5.1 Definition of the sustainability coefficient of the funded system

T I_j, j

τ_1

τ_1

τ_2 I_j τ_2

τ_2

S_j I_j λ_j

a_j EUR i

$$\tau_2 = \frac{S_j(1+i)^{T-1} + S_j(1+i)^{T-2}(1+\lambda_j) + \ldots + S_j(1+\lambda_j)^{T-1}}{a_j(1+i)^{T-1} + \ldots + a_j}$$

$$= \frac{S_j}{a_j(i-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}.$$

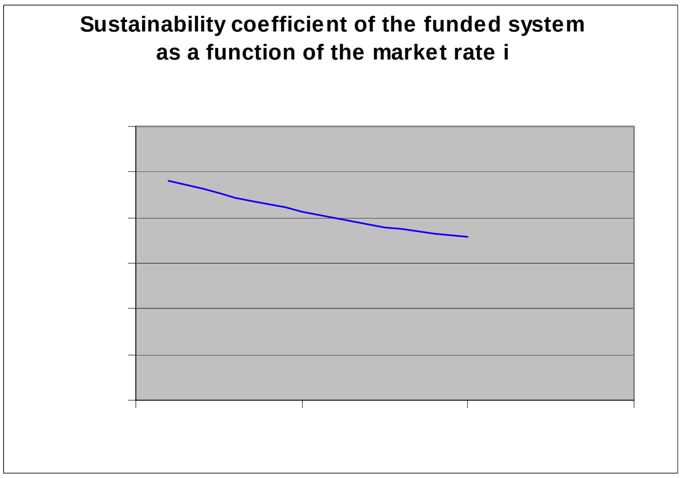


FIG 6: SUSTAINABILITY COEFFICIENT OF THE FUNDED SYSTEM AS A FUNCTION OF THE MARKET RATE i

τ_2 I

τ_2 $E(\tau_2)$ $\mathrm{Var}(\tau_2)$ a_j

a_j^2

$$E(\tau_2) = \frac{c}{a_j}$$

$$\mathrm{Var}(\tau_2) = \frac{K}{a_j^2}.$$

5.2 Definition of the overall sustainability coefficient

x

$\tau(x)$

x *(1-x)*

$$\tau(x) = x\tau_1 + (1-x)\tau_2,\ x \in [0,1].$$

τ_1 τ_2 $\tau(x)$

$E\left[\tau(x)\right] = xE(\tau_1) + (1-x)E(\tau_2)$ $\mathrm{Var}\left[\tau(x)\right] = x^2\mathrm{Var}(\tau_1) + (1-x)^2\mathrm{Var}(\tau_2)$

5.3 The sustainability of the system

a

U = U (a)

$U'(a) \le 0$

$$G(x) = \frac{\mathrm{Var}(\tau_1) - \mathrm{Var}\left[\tau(x)\right]}{\mathrm{Var}(\tau_1)} \ge G^* \qquad (1)$$

$x = x^*$ *U = U (a)*

G^*

l

T

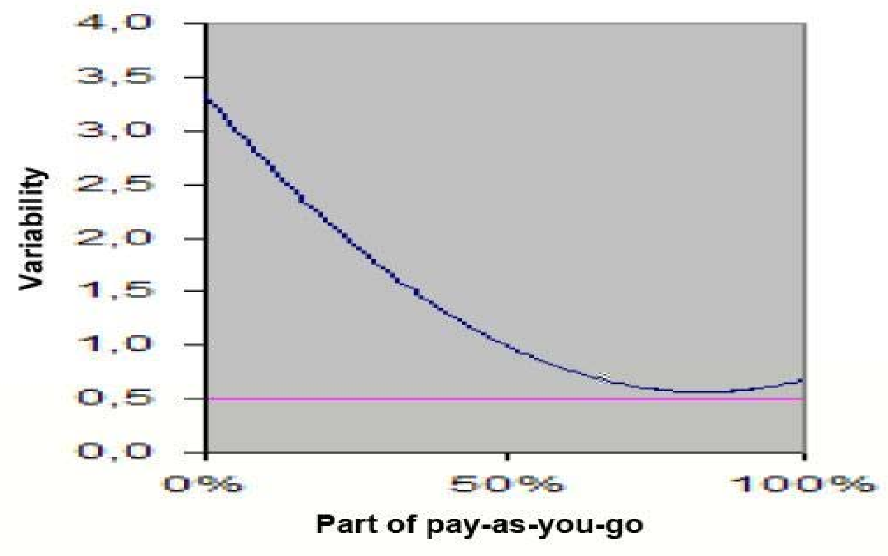
$\tau(x)$ *x* T

l

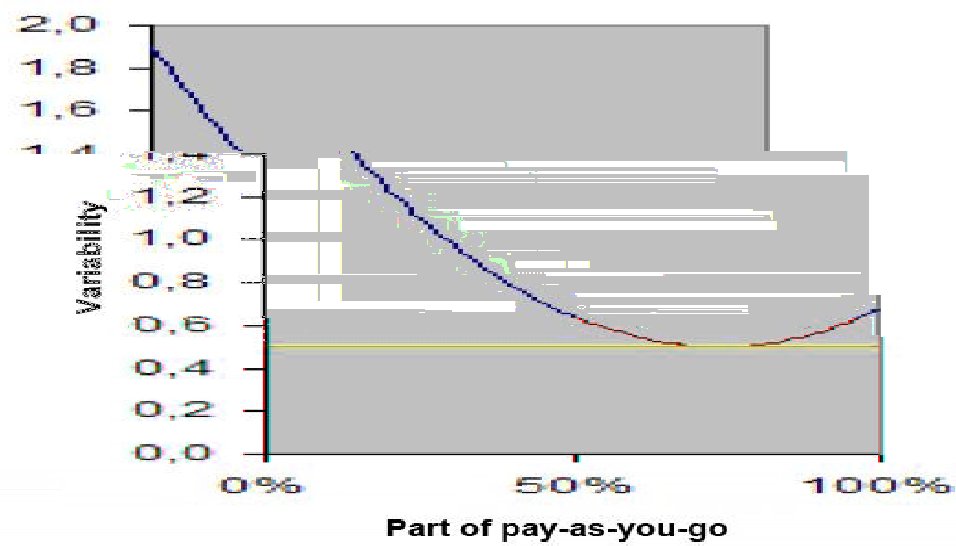
l

T

With an annuity of a = 2000 €, it is impossible to reduce the risk of variabilty to the threshold of 0.5 .



With an annuity of a = 2650 €, it is possible to reduce the risk of variabilty to the threshold of 0.5 .



$$K = \mathrm{Var}\left[\frac{S_j}{(1-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}\right].$$

$$\mathrm{Var}\left[\tau(x)\right] \le (1-G^*)\mathrm{Var}(\tau_1),$$

$$\mathrm{Var}\left[\tau(x)\right] = x^2\mathrm{Var}(\tau_1) + (1-x)^2\mathrm{Var}(\tau_2) = x^2\mathrm{Var}(\tau_1) + (1-x^2)\frac{K}{a^2} \le \mathrm{Var}(\tau_1)(1-G^*),$$

$$(K + a^2Var(\tau_1))x^2 - 2Kx - a^2Var(\tau_1)(1-G^*) + K \le 0 \qquad (2)$$

$$\min\left[(K + a^2\,\mathrm{Var}(\tau_1))\,x^2 - a^2 - 2Kx\,\mathrm{Var}(\tau_1)\,(1\text{-}G^*) + K\right] \le 0$$

$$\text{-}a^2Var(\tau_1)\frac{(a^2\mathrm{Var}(\tau_1)\text{ - }G^*K - G^*a^2\mathrm{Var}(\tau_1)}{K + a^2Var(\tau_1)} \le 0,$$

$$x = \frac{\mathrm{K}}{K + a^2Var(\tau_1)}.$$

$$y = (K + a^2Var(\tau_1))x^2 - a^2 - 2KxVar(\tau_1)\,(1-G^*) + K$$

$$a^2 \ge \frac{G^*K}{\mathrm{Var}(\tau_1)(1-G^*)}$$

$U(a)$

$$a^* = \sqrt{\frac{G^*K}{\mathrm{Var}(\tau_1)(1-G^*)}}.$$

$$\min[(K + a^2\mathrm{Var}(\tau_1))x^2 - 2Kx - a^2\mathrm{Var}(\tau_1)(1-G^*) + K] = 0.$$

$$x^* = \frac{K}{K + a^2Var(\tau_1)} = \frac{K}{K + \frac{G^*K}{Var(\tau_1)(1-G^*)}\sigma_1^2} = 1 - G^*.$$

5.4 Conclusion: the optimal annuity

$$x = x^*$$

$$G(x) = \frac{\mathrm{Var}(\tau_1) - \mathrm{Var}[\tau(x)]}{\mathrm{Var}(\tau_1)} \geq G^*$$

$x^* = 1 - G^*$

$$\mathrm{a}^* = \sqrt{\frac{G^* K}{\mathrm{Var}(\tau_1)(1-G^*)}},$$

$$K = \mathrm{Var}\left[\frac{S_j}{(1-\lambda_j)}\right] i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}$$

a^* x^*

$$G^* = \frac{\mathrm{Var}(\tau_1) - \mathrm{Var}\left[\tau(x)\right]}{\mathrm{Var}(\tau_1)}$$

and

$$1 - G^* = \frac{\mathrm{Var}(\tau_2) - \mathrm{Var}\left[\tau(x)\right]}{\mathrm{Var}(\tau_2)}.$$

	4466	713	1448	5231	220	6364	2809	743	3140

Var $[\tau(x)] \leq \mathrm{Var}(\tau_1) = 0.68$

Var $[\tau(x)] = 0.34$

6. ANALYSIS OF THE RESULTS

d	sustainability index

	9,7%	2,9%	4,4%	8,9%	0,6%	8,1%	3,9%	1,4%	3,3%

	51,7%	90,2%	80,7%	55,3%	99,6%	59,2%	84,2%	97,3%	87,7%

$$K = \mathrm{Var}\left[\frac{S_j}{(1-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}\right].$$

G^*

$$a^* \geq \sqrt{\frac{K}{Var(\tau_1)}}$$

G^* x^*

G^*

	48,3%	9,8%	19,3%	44,7%	0,4%	40,8%	15,8%	2,7%	12,3%

$$x^* = 1 - G^*$$

G^*

7. BIBLIOGRAPHY