Voting on pensions: sex and marriage.

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Abstract

Existing political economy models on pensions focus on age and productivity. In this paper we focus on two other individual characteristics: sex and marital status. We assume away age (people vote at the start of their life) and thus look at the most preferred rate of taxation that finances a Beveridgean pension of individuals that are characterised by a certain wage rate, sex and marital status. We also allow for two types of couples: either one-breadwinner or two-breadwinner couples. Marriage pools both wage and longevity differences between men and women. Hence singles tend to have more extreme most preferred tax rates than couples. We show that the majority voting solution depends on the relative number of one-breadwinner couples and on the size of derived pension rights.

Keywords: social security, differential longevity, majority voting, individualisation of pension rights.
1 Introduction

If there were no limit to the length of a title, we would have entitled our paper: Why do men consistently agree with pension schemes that penalize them? One indeed knows that most pension systems provide benefits that are longevity-invariant and sometimes contribution-invariant. Given that men have a shorter life expectancy than women and earn and thus contribute more than women, it is clear that such pension systems are to their detriment. The first answer one can offer to this question is trivial: women outnumber men and thus can impose their view. Another answer is that with flat rate benefits low-income men can back such schemes granted that earnings differences dominate longevity differences. Yet the best answer might be that in a society where a majority of men and women are married longevity and earnings differences are pooled within the couple and this makes any sex war irrelevant.

Women live longer than men and they earn less than men on average. For instance, in France, it is estimated that women life expectancy at 60 is 20% higher than that of a man and that the pay gap is around 20%. At the same time, one also knows that low-income people, men and women, have a lower longevity than high-income people. This has lead to studies showing that social security systems that look redistributive but provide longevity-invariant benefits, are in fact not that redistributive (see e.g. Coronado et al. 2000, Liebman 2001 and Bommier et al. 2006). Taking just one example, Bommier et al. (2006) estimate that the French public pension system redistribution is reduced by up to 50% because it is longevity invariant.

Not only Social Security operates redistribution among individuals with different income and longevity, but, as shown by Galasso (2002), redistribution by marital status is also surprisingly large. Indeed, he showed that one-earner couples get the highest internal return from the Social Security, followed by two-earner couples with 70/30 earnings split; returns are equal for two-earner couples with a 50/50 earnings split and single women while single men are the most disadvantaged. The difference in returns observed between married couples, either one-earner or two-earner, and singles can be explained by the so-called “derived pension rights”. For instance, several countries, like France, provide the surviving spouse (more often the woman) with a survivor benefit, while some other countries provide one-earner couples with a higher replacement rate than the one of a single man; some countries, like Belgium or Japan, provide both types of derived benefits. The marital status and the generosity of the system toward the non working spouse is

\[1\text{For example, in Belgium, the supplementary pension is evaluated to } 1/4 \text{ of the working spouse pension. As shown in Gruber and Wise (1999), derived pension rights may take very different forms depending on the country.}\]
then likely to play an important role in the support for a pension system.

Over the last decades one has observed an interesting evolution regarding pensions. Women are more and more frequently going into paid employment and pension rights are increasingly individualized, which implies an eventual abolition of derived rights. One can expect that this dual evolution has some incidence on the pension system determined by the political process.

There exists a number of political economy papers trying to explain existing pension systems using majority voting models. The first one by Browning (1985) focuses on age differences: the old being in favour of generous pensions and the young preferring private saving, the decisive voter is the median age one. A second generation of models introduces differences not only in age but also in wage rates. For example, Casamatta et al. (2000) show that the pension system is chosen by a majority made of rich and poor workers who collude against a coalition of retirees and middle class workers; this is the so-called *ends against the middle* outcome.

In this paper we want to focus on the majority chosen pensions in a society where a majority of men and women are married and a minority is single. Men live shorter and earn more than women. Assuming that retirement consumption is financed by the proceeds of private saving and by a Beveridgean pension, we want to test several hypotheses:

- what is the effect of longevity and wage gender gap on the chosen tax rate?
- what is the effect of increasing the number of married couples on the size of the pension system?
- what is the effect of increasing the relative number of one-breadwinner (versus two-breadwinners) couples on the pension system?
- what is the effect of the individualization of pension rights (equivalently, the reduction of derived pension rights) on the pension scheme?

These questions are certainly relevant when it comes to the prevailing pension schemes and surprisingly they have hardly been addressed in the literature. In particular, the generosity of the system toward non-working spouses may play an important role on the political support of one-breadwinner couples towards existing pensions. This is a timely issue. More and more

\(^2\)For good surveys, see Galasso and Profeta (2002) and de Walque (2005).

\(^3\)See however Borek (2007) and Leroux (2008) who have introduced longevity differentials in political economy models of social security.

\(^4\)For a good survey on the role of derived pension rights on old-age income security of women in OECD countries, see Choi (2006).
countries abandon the “derived pension rights model” and prefer instead, the so-called “adult worker model”. For instance, Denmark has suppressed survivor benefits and Germany has moved toward a “family splitting” system and provide a compensation for interrupted careers (i.e. a pension credit per child).5

The setting we adopt is standard. People live for two periods, work in the first one and retire in the second. They control two variables: their private saving and the payroll tax rate through voting. To keep the analysis tractable, we make a number of assumptions, like quasi-linear utility function, no liquidity constraint and certain length of life. We assume that individuals vote at the beginning of their life. All men have the same longevity, which is lower than that of women. Men and women have the same productivity but that the wage of the woman is only a fixed fraction of the one of the man. We also assume positive assortative mating, i.e. men marry women who have the same wage as theirs up to that fraction.6 Finally, the pension system is Beveridgean so that pension benefits and payroll tax rates are uniform.

Under this framework, low-productivity and high-longevity individuals support the existence of a pension system. Thus, single women who have smaller wages and higher longevity will be in favour of a pension system while single men who have higher wages and smaller longevity will be against it. We further introduce couples. When the couple comprises two breadwinners, it neutralizes gender differences in wages and in longevity so that they get a zero net benefit from the pension system; in this case, they are indifferent between public pensions and private savings, as a mean to transfer resources between periods. However, because the labour supply is endogenous in our setting, a pension system creates labour distortions so that they end up preferring a zero tax rate. On the contrary, one-breadwinner couples do not neutralize gender differences and they will be in favour of a pension system only if derived pension rights are important and sufficient to counterbalance the husband’s net contribution to the pension system. Thus, the support for a pension system will depend both on the number of one-breadwinner couples and on the generosity of the pension system. We further extend our model and allow for a productivity distribution. Our results are robust to this new specification; the only difference is that now, in each type of households, some individuals (the ones at the bottom of the productivity distribution) are always in favour of a pension system, because of the amount of income redistribution they obtain.

5On this, see Choi (2006), Veil (2007) and Bonnet and Geraci (2009).
6The papers of Mare (1991), Pencavel (1998) and Qian (1998) find strong evidence of positive assortative mating with respect to education. Education can be regarded as a good proxy for income.
Finally, our model predicts that the recent trend toward the individualization of pension rights should lead to reduced payroll tax levels. On the contrary, our model does not give clear conclusions concerning the evolution of the number of two-breadwinner couples. Depending on this number and on the generosity of the system, this should lead to an increase or a decrease in the tax rate.

The rest of the paper is organized as follows. Section 2 presents a standard political economy model with a double heterogeneity: wages and longevity. Section 3 introduces gender and marriage. In Section 4, we add productivity differences and in the last Section we discuss the assumptions made in our model and some possible extensions.

2 The basic model

In this model, we assume that individuals live at most for two periods. They work in the first period and retire in the second one. Each individual of type \(i\) is characterized by a pair \((w_i, \pi_i)\) where \(w_i\) is the labor productivity in the first period and \(\pi_i\) is the length of the second period of life.7

The intertemporal utility function of any individual of type \(i\) is quasi-linear in the first period consumption and is represented by

\[
u_i (c_i, d_i, l_i) = c_i - v(l_i) + \pi_i u(d_i)
\]

where \(c_i\) and \(d_i\) denote the first and second period consumptions respectively and \(l_i\) is labor supply. Second-period utility function, \(u(.)\) is such that \(u'(.) > 0\) and \(u''(.) < 0\). The disutility of labor, \(v(l_i)\), is quadratic and equal to \(l_i^2/2\). In our model, individuals work, contribute to the pension system, consume and save in the first period. In the second period, they retire and receive a pension benefit \(p\). We also assume a perfect annuity market and a zero interest rate so that the return on savings is simply \(1/\pi_i\). First and second period consumptions can then be written as

\[
c_i = (1 - \tau) w_i l_i - s_i
\]

\[
d_i = \frac{s_i}{\pi_i} + p
\]

where \(\tau \in [0,1]\) is the payroll tax rate and \(s_i\) is the amount of savings.

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7Allowing for uncertain mortality would be possible (\(\pi_i\) being the probability of surviving the second period) but more complicated. We believe that it would not modify substantially our conclusions.
Throughout the paper, we assume away liquidity constraints so that \( s_i \) can be positive as well as negative. The problem of type \( i \)'s individual consists in solving

\[
\max_{c_i, l_i} \quad c_i - l_i^2/2 + \pi_i u(d_i)
\]
\[
\text{s.t.} \quad \begin{cases} 
  c_i = (1 - \tau) w_i l_i - s_i \\
  d_i = \frac{s_i}{\pi_i} + p
\end{cases}
\]

From the first order conditions we obtain:

\[
l_i^* = (1 - \tau) w_i \\
u'(d_i^*) = 1
\]

As to the pension system, we assume that individuals contribute to the pension system during the first period of their life and receive a flat pension benefit in the second period (i.e. the retirement period). Thus a feasible pension system must satisfy the following budget constraint,

\[
p \sum n_i \pi_i \leq \sum \tau n_i w_i l_i^*,
\]

where \( n_i \) denote the relative number of individuals of type \( i \). Note that here, \( p \) is an annual pension benefit which implies that a person who lives long gets in total more than a person who has a short life. Under the assumption of perfect budget balance, the expression of the pension benefit is

\[
p(\tau) = \tau \frac{(1 - \tau) Ew^2}{\bar{\pi}}, \tag{1}
\]

where \( Ew^2 \) is the average square productivity. Every individual contributes an amount which is proportional to his labour income and receives a uniform pension benefit during a retirement period of unequal length \( \pi_i \). Such a pension system redistributes resources from high-productivity toward low-productivity individuals and from short-lived toward long-lived individuals.\(^8\)

The indirect utility function of an individual of type \( i \) is then

\[
V^i(\tau) = \frac{(1 - \tau)^2 w_i^2}{2} - s_i^* + \pi_i u \left( \frac{s_i^*}{\pi_i} + p(\tau) \right) \tag{2}
\]

where the star stands for the optimal level. The preferred tax rate of this individual is obtained by solving the following program:

\[
\max_{\tau \in [0,1]} V^i(\tau)
\]

\(^8\)Most PAYG pension schemes exhibit such features. On this topic, see for example, Coronado et al. 2000, Liebman 2001 and Bommier et al. 2006.
In appendix, we show that the solution to this problem is

\[
\tau_i^* = \begin{cases} 
0 & \text{for } \frac{w_i^2}{Ew^2} \geq \frac{\pi_i}{\bar{\pi}} \\
\frac{\pi_i Ew^2 - w_i^2}{\bar{\pi} Ew^2 - w_i^2} & \text{otherwise}
\end{cases}
\]  

(3)

The level of the tax rate chosen by the individual depends on the level of expected redistribution he gets from the pension system. There are two possible ways he may gain from the pension system: either because he has a longer life duration than the average or because, he has a lower productivity than the average. Hence, the preferred tax rate of any individual will be zero if he has characteristics such that \( \frac{w_i^2}{\pi_i} \geq \frac{Ew^2}{\bar{\pi}} \). It is clear that the lower the wage rate and the higher the longevity the more likely an individual will be in favor of the pension scheme. The equality \( \frac{w_i^2}{\pi_i} = \frac{Ew^2}{\bar{\pi}} \) gives us the separating locus of types which divides those who are in favor and those who are against the tax. In the Figure 1, we represent this function in the plane \((w_i, \pi_i)\)

![Figure 1](image)

Shape of \( \pi_i = \frac{w_i^2 \bar{\pi}}{Ew^2} \)

To the left of the curve, one finds the types who are in favor of a positive tax; to the right, they are against. One can also note that, when positive, the most preferred tax decreases with \( w_i \) and increases with \( \pi_i \). Take the case of someone with a zero wage, his most preferred tax rate is equal to 1/2 and not 1. This comes from the efficiency cost of taxation: 1/2 is the tax that provides the maximum revenue, i.e. the peak of the Laffer curve.
Majority voting when there are two characteristics raises some technical problems that will be solved below by assuming a particular relation between the two characteristics. For the time being assume that all individuals have the same longevity \( \pi_i = \bar{\pi} \) and that the wage rate have the standard density function with median wage below average wage: \( \bar{w} \geq w_m \). We know from Jensen inequality that \( \sqrt{Ew^2} > \bar{\pi} \). Given that in the relevant range of \( w \), the most preferred tax decreases with \( w \), the Condorcet winners are the individuals with a median wage.

3 A model with a unique productivity level.

We now assume that individuals are characterized by different gender and start with a society consisting of only singles. We then introduce the possibility of marriage and allow these couples to comprise either one breadwinner or two breadwinners.

We have the same number of men and women. These are characterized by a pair \((\pi, w)\) for men and by a pair \((\pi_f, w_f)\) for women such that

\[
\pi_f = \beta \pi \\
w_f = \alpha w
\]

with \( \alpha \leq 1 \) and \( \beta \geq 1 \).

In other words, we posit that women always have a higher life duration than men but also face lower wage. Note that in this section, we assume a unique productivity level, \( w \) for men and \( \alpha w \) for women.

In this case, the pension benefit (1) is now equal to

\[
p(\tau) = \tau (1 - \tau) \frac{(1 + \alpha^2) w^2}{(1 + \beta) \bar{\pi}}
\]

3.1 The political equilibrium in a society of singles

Under our assumptions on genders, we obtain from expression (3) that the preferred tax rates for men and women are respectively:

\[
\tau_M^* = 0 \quad \text{as} \quad \frac{w^2}{Ew^2} = \frac{2}{1 + \alpha^2} \geq \frac{\pi}{\bar{\pi}} = \frac{2}{1 + \beta}
\]

\[
\tau_F^* = \frac{\beta \pi}{\frac{\beta}{\bar{\pi}} Ew^2 - \alpha^2 w_f^2} \quad \text{as} \quad \frac{\alpha^2 w_f^2}{Ew^2} = \frac{2\alpha^2}{1 + \alpha^2} \leq \frac{\beta \pi}{\bar{\pi}} = \frac{2\beta}{1 + \beta}
\]

\footnote{For simplicity, we restrict attention to the most realistic case, where \( \alpha \leq 1 \) and \( \beta \geq 1 \) but the analysis could be extended to \( \alpha > 1 \) and \( \beta < 1 \).}

\footnote{In Section 4, we relax this assumption.}
Thus a man, who has lower longevity and higher productivity than the average, always prefers a zero tax rate since he is always a net contributor to the pension system. On the contrary, a woman always gets a net benefit from the pension system and votes for a positive tax rate. The political equilibrium corresponds to the preferred tax rate of the median individual. Hence, assuming a slightly higher number of women than men, the political outcome in a society which comprises only singles is the preferred tax rate of a woman and is equal to $\tau^* = \tau^*_F$.

### 3.2 The political equilibrium in a society comprising both singles and couples

We now model the decisions made by a couple. To this purpose, we assume that spouses play cooperatively and share their resources over their life-cycle. We further assume positive assortative mating so that a man with productivity $w$ always gets married with a woman with productivity $\alpha w$. A two-breadwinner couple thus solves the following problem:

$$\max_{c, d, l_m, l_f} 2c - \frac{l_f^2}{2} - \frac{l_m^2}{2} + (\pi_f + \pi_m) u(d)$$

s.t. $(wl_m + w_f l_f) (1 - \tau) + (\pi_f + \pi) p \geq 2c + (\pi_f + \pi) d$

where $d$ represents the individual level of consumption in the second period for each member of the couple. Because of the quasi-linearity in consumption, the labour supply for the husband and the wife are respectively $l^*_m = w(1 - \tau)$ and $l^*_f = \alpha w (1 - \tau)$. Note, that under our assumptions, these are independent of their marital condition (whether they live as a couple or whether they are single). Hence, the woman labour supply is always lower than that of a man. This implies that her total contributions to the pension system $(\alpha w \tau)$ are also lower while they receive a higher total pension benefit $\beta \pi p \geq \pi p$.

Substituting for $l^*_m$ and $l^*_f$ and for the expression $p(\tau)$ of the pension benefit, we obtain the couple’s indirect utility function

$$V^{c2}(\tau) = \frac{(1 - \tau)^2}{2} (1 + \alpha^2) w^2 - (1 + \beta) \pi d^* + (1 + \beta) \pi p(\tau) + (1 + \beta) \pi u(d^*)$$

where $c2$ stands for a couple with two breadwinners. Note that the pension benefit expression is not modified by the introduction of two-breadwinner couples as both members contribute to and benefit from the pension system in the same way as if they were singles. Differentiating this indirect utility function with respect to the tax rate $\tau$, it is straightforward to show that the
preferred tax of a two-breadwinner couple is always nil, $\tau^*_{c2} = 0$. Note that if the labour supply were exogenous, the couple would be indifferent between any level of taxation (it would obtain the same return from savings as from the pension system); when labour supply is endogenous, the preferred tax rate is zero as in this case, the pension system introduces distortions on the labour supply (the return from the pension system is smaller than the return from private savings).

Let now consider the political equilibrium and assume that a fraction $\varphi$ of men and women get married. The preferred tax rates remain the same for single women and men, since the existence of couples does not modify the expression of the pension benefit so that $\tau^*_M = 0$ and $\tau^*_F > 0$ (as defined before) while $\tau^*_{c2} = 0$. In this case, there is a majority of individuals in favour a zero tax rate so that the political outcome will be $\tau^* = 0$. If the labor supply were exogenous, the couple would be indifferent between any level of taxation and we would then have had exactly the reverse result, i.e. a maximum tax rate, $\tau^* = \tau^*_F = 1$ (under the assumption of a slightly higher number of women than of men).

### 3.3 Introducing one-breadwinner couples

#### 3.3.1 The modified model

Let now assume that society consists of four different categories of households: single men, single women, couples with one breadwinner and couples with two breadwinners. As in the previous sections, there is still an equal fraction $(1 - \varphi)$ of single males and of single females and a fraction $\varphi$ of couples, so that a number $2\varphi$ of individuals live in couple. But, we now assume that, among these couples, a fraction $\mu$ is composed of two breadwinners, while a fraction $(1 - \mu)$ of couples consists of only one breadwinner.$^{11}$

This breadwinner is always the husband and his wife may be entitled to a certain benefit, even though she did not contribute to the pension system. These benefits, sometimes called derived pension rights, consist of a small supplementary pension plus a survival pension. We thus assume that she receives a fraction $\gamma \in [0, 1]$ of the full per period pension benefit $p(\tau)$ during the second period of her life of length $\pi_f.$ $^{12}$ If $\gamma = 0$, the spouse receives

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$^{11}$Note that Section 3.1 is equivalent to assuming $\varphi = 0$, while Section 3.2 corresponds $\varphi \in [0, 1], \mu = 1$.

$^{12}$This parameter $\gamma$ may account either for a survivor benefit or for the higher replacement rate provided to a one-earner couple than to a single individual. As mentioned in the introduction, such features are observed in many countries with a public pension scheme.
nothing in the second period while if $\gamma = 1$, she gets a full pension. Whatever the value of $\gamma$, yearly consumption is the same for both spouses.

Let us first define the problem of a one-breadwinner couple. This problem is slightly different from the case with two breadwinners (i.e., problem $A$):

$$\max_{c,d,l_m} 2c - l_m^2/2 + (\pi_f + \pi_m) u(d) \tag{B}$$

s.t. $wl_m (1 - \tau) + (\gamma \pi_f + \pi) p \geq 2c + (\pi_f + \pi) d$

Only the man supplies labour and $l_m^* = w(1 - \tau)$. Substituting for $l_m^*$ and $\pi_f$, the indirect utility function is equal to:

$$V^{cl} (\tau) = \frac{w^2 (1 - \tau)^2}{2} + (1 + \gamma \beta) \pi p (\tau) - (\beta + 1) \pi d^* + (\beta + 1) \pi u (d^*) \tag{6}$$

Note that the expression $p(\tau)$ is now modified by the existence of one-breadwinner couples. To see this clearly, we rewrite the budget constraint as

$$p[\pi + \pi_f (1 - \varphi + \varphi \mu + \gamma \varphi (1 - \mu))] \leq wl_m^* \tau + w_f l_f^* (1 - \varphi + \varphi \mu)$$

On the left hand side, we have total benefits distributed, i.e., every working individual (men or women) receive a pension $p$ and a fraction $\varphi (1 - \mu)$ of women receive $\gamma p$. On the right hand side, we have total contributions; in this model, whatever their marital situation, men supply labour, but only single women and women belonging to a two-breadwinner couple supply labour and thus pay contributions. Replacing for optimal labour supplies, total contributions are equal to $(1 - \tau) \tau [1 + \alpha^2 (1 - \varphi + \varphi \mu)] w^2$. Substituting for $\pi_f = \beta \pi$ on the left hand side and using the budget balance condition, we obtain a new expression for the flat rate pension benefit,

$$p(\tau) = \frac{(1 - \tau) \tau [1 + \alpha^2 (1 - \varphi + \varphi \mu)]}{\pi [1 + \beta (1 - \varphi + \varphi \mu + \gamma \varphi (1 - \mu))]} w^2. \tag{8}$$

Note that if we assume a society of only singles ($\varphi = 0$) or a society of couples with two breadwinners ($\varphi = 1$ and $\mu = 1$), $p(\tau)$ is equal to (4) as before. In this case, we are back to the previous subsections.

For the following sections and in order to simplify notations, we define here the function $\chi(\alpha, \beta, \varphi, \mu, \gamma)$:

$$\chi(\alpha, \beta, \varphi, \mu, \gamma) = \frac{[1 + \alpha^2 (1 - \varphi + \varphi \mu)]}{[1 + \beta (1 - \varphi + \varphi \mu + \gamma \varphi (1 - \mu))]} \tag{7}$$

so that the “modified” pension benefit can be rewritten as

$$p(\tau) = \frac{(1 - \tau) \tau}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2. \tag{8}$$
3.3.2 Preferred tax rates and the political equilibrium

Individuals preferred tax rates are obtained by solving

$$\max_{\tau \in [0,1]} V^i(\tau)$$

where $i$ accounts for $M$ (single male), $F$ (single female), $c_2$ for two-breadwinner couples and $c_1$ for one-breadwinner couples. For one breadwinner couples, the indirect utility function is equal to (6) while for the other households, indirect utility functions remain the same and defined by (2) and by (5); only the expression $p(\tau)$ is modified. In appendix B, we derive the solution for each type of individuals and find that the preferred tax rates are equal to

$$\tau^*_M = \tau^*_{c_2} = 0$$

$$\tau^*_F = \begin{cases} 
0 & \text{if } \gamma > \hat{\gamma}_F = \beta - \alpha^2 \frac{1}{\beta \alpha^2 \varphi (1 - \mu)} \\
\frac{\beta \chi(a, \beta, \varphi, \mu, \gamma) - \alpha^2}{2 \beta \chi(a, \beta, \varphi, \mu, \gamma) - \alpha^2} & \text{if } \gamma < \hat{\gamma}_F
\end{cases}$$

$$\tau^*_{c_1} = \begin{cases} 
0 & \text{if } \gamma < \hat{\gamma}_{c_1} = \frac{\beta - \alpha^2}{\beta (1 + \alpha \gamma (1 - \varphi + \varphi \mu) + \varphi (1 - \mu))} \\
\frac{(1 + \beta \gamma) \chi(a, \beta, \varphi, \mu, \gamma) - 1}{2 (1 + \beta \gamma) \chi(a, \beta, \varphi, \mu, \gamma) - 1} & \text{if } \gamma > \hat{\gamma}_{c_1}
\end{cases}$$

where $\hat{\gamma}_F$ and $\hat{\gamma}_{c_1}$ are the threshold levels of $\gamma$ for which the preferred tax rates become strictly positive. As before, the preferred tax rates of single men and of two-breadwinner couples are zero; their preference for a zero tax rate is here reinforced by the fact that the pension system now also operates redistribution toward one-breadwinner couples. For single women and one-breadwinner couples, the level of their preferred tax rate depends on the level of $\gamma$, i.e. on the level of generosity of the system toward one-breadwinner couples. Indeed, women prefer a strictly positive tax rate only when the system is not too generous toward the non-working spouse, since more redistribution to the latter is always to the detriment of single women (they get less from the pension system). For one-breadwinner couples, it is the contrary, they will always prefer a strictly positive tax rate if the system is sufficiently redistributive toward them. If $\gamma \to 0$, the man in the couple contributes to a system which is not favorable to him (since he has higher productivity and lower longevity). In this specific case, he obtains almost no survivor benefit compensation so that he votes for a zero tax rate. On the contrary, if $\gamma$ is high, his net contribution to the pension system can be compensated by the outside benefit given to his non-working spouse.

We now determine the political equilibrium level of the tax rate. It depends on the generosity of the system toward one-breadwinner couples, i.e.
on the level of $\gamma$. In appendix, we show that $\hat{\gamma}_{c1} < \hat{\gamma}_F$. We obtain three possible cases:

- if $\gamma < \hat{\gamma}_{c1} < \hat{\gamma}_F$: individuals preferred tax rates are $\tau^*_{M} = \tau^*_{c2} = \tau^*_{c1} = 0$ and $\tau^*_{F} > 0$. In this case, a majority of individuals prefer a zero tax rate: $\tau^* = 0$.

- if $\hat{\gamma}_{c1} < \gamma < \hat{\gamma}_F$: $\tau^*_{M} = \tau^*_{c2} = 0$ and $\tau^*_{c1} > 0, \tau^*_{F} > 0$. If $\mu < 0.5$, the equilibrium tax rate should be positive. In appendix, we show that

\[
\begin{align*}
\tau^*_{F} &< \tau^*_{c1} \text{ iff } \gamma > \frac{\beta - \alpha^2}{\beta \alpha^2} = \hat{\gamma} \\
\tau^*_{c1} &> \tau^*_{c1} \text{ iff } \gamma < \hat{\gamma}
\end{align*}
\]

If $\gamma$ is not too high, i.e. $\hat{\gamma}_{c1} < \gamma < \hat{\gamma}$, the chosen tax rate is likely to be the one preferred by one-breadwinner couples. On the contrary, if $\hat{\gamma} < \gamma < \hat{\gamma}_F$, the chosen tax rate is the one preferred by single women.

This case corresponds to the traditional image of couples with male breadwinners and non working housewives who benefit from generous derived rights.

- Finally, if $\hat{\gamma}_{c1} < \hat{\gamma}_F < \gamma$, only one-breadwinner couples vote for a positive tax rate, $\tau^*_{c1} > 0$ while other categories can form a coalition and vote for a zero tax rate so that the political equilibrium is most likely $\tau^* = 0$ (except if the number of single women and one breadwinner couples have the majority).

To sum up, the existence of one-earner couples (and their relative number) as well as the generosity of the system toward them (through the level of the parameter $\gamma$) certainly influences the level of the tax rate chosen at the voting equilibrium.

In the next section, we extend our model so as to take into account differences in productivities, not only between genders but also across individuals in general.

4 A model with a distribution of productivity

In this section, we keep the assumption that individuals’ longevity can take only two values (i.e. $\pi$ and $\pi_f$ for men and women respectively). In contrast, we now assume that $w$ is uniformly distributed, with support $[0, 1]$. The average and the median productivity are then identical and equal to $\bar{w} =$
$w_m = 1/2$. We further assume that $w_f$ is distributed over $[0, \alpha]$ with density, $1/\alpha$.\textsuperscript{13} With the results obtained above in mind, we expect that now every individuals at the bottom of the productivity distribution will be in favour of a pension system as they will benefit from the redistributive pension system. Thus, independently of their marital status and gender, a fraction of individuals (in every type of households) will be vote for a positive tax rate. In contrast individuals at the top of the productivity distribution will be against a pension system. Yet, as the two following graphs intend to show, depending on which household category they belong to, more or less agents will be against it. At the same time we expect that the treshold productivity that separate those in favor a positive pension and those against will vary across our three types. It will be high for single men and two-breadwinners couples and low for single men and one-breadwinner couples when $\gamma$ is high enough. Figure 2 present two profiles that correspond to an example given below. The sum of the 4 histograms is equal to 1, that is half of our normalized population.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Figure 2}
\end{figure}

\subsection{Individuals preferred tax rates}
When $w$ follows a uniform distribution, the pension benefit becomes

$$p(\tau) = \frac{1}{3} \frac{\tau (1-\tau)}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma)$$

\textsuperscript{13}We could equally have assumed a right-skewed distribution, but this would have complicated our model without providing additional results.
where 1/3 corresponds to the average square productivity. In appendix, we show that individuals preferred tax rates are now

\[
\tau^*_F(w) = \begin{cases} 
0 & \text{if } \omega^2 > \beta \chi(\alpha, \beta, \varphi, \mu, \gamma) \\
\frac{\beta \chi(\alpha, \beta, \varphi, \mu, \gamma) - \omega^2}{2 \omega^2 \alpha^2 - \omega^2} & \text{otherwise} 
\end{cases} \tag{12}
\]

\[
\tau^*_M(w) = \begin{cases} 
0 & \text{if } \frac{w^2}{1/3} > \chi(\alpha, \beta, \varphi, \mu, \gamma) \\
\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma) - w^2}{2 \omega^2 \alpha^2 - w^2} & \text{otherwise} 
\end{cases} \tag{13}
\]

\[
\tau^*_{2c}(w) = \begin{cases} 
0 & \text{if } \frac{w^2}{1/3} > \frac{(1+\beta)(1+\alpha^2)}{1+\beta}(1+\alpha^2) \chi(\alpha, \beta, \varphi, \mu, \gamma) \\
\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma) - w^2}{2 \omega^2 \alpha^2 - w^2} & \text{otherwise} 
\end{cases} \tag{14}
\]

\[
\tau^*_{1c}(w) = \begin{cases} 
0 & \text{if } \frac{w^2}{1/3} > \frac{(1+\gamma \beta)(1+\alpha^2)}{1+\gamma \beta}(1+\alpha^2) \chi(\alpha, \beta, \varphi, \mu, \gamma) \\
\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma) - w^2}{2 \omega^2 \alpha^2 - w^2} & \text{otherwise} 
\end{cases} \tag{15}
\]

where the \( w \) concerns the productivity of the individuals concerned: M, F, 2c, 1c.

4.2 Political equilibrium

In order to determine the political equilibrium, we manipulate expressions (12), (13), (14) and (15) so as to obtain the wage rate as a function of the most preferred tax rate, instead of the other way round. Since we have assumed a uniform distribution, this also gives us the number of individuals who prefer this tax rate (or a greater one) to any other level of the tax rate. This yields:

\[
w_M(\tau) = \sqrt{\frac{1 - 2\tau \chi(\alpha, \beta, \varphi, \mu, \gamma)}{1 - \tau}} \frac{1}{3}
\]

\[
w_F(\tau) = \sqrt{\frac{\beta}{\alpha^2}} w_M(\tau)
\]

\[
w_{2c}(\tau) = \sqrt{\frac{1 + \beta}{1 + \alpha^2}} w_M(\tau)
\]

\[
w_{1c}(\tau) = \sqrt{1 + \gamma \beta} w_M(\tau)
\]

It is straightforward to show that we always have \( w_M(\tau) < w_{2c}(\tau) < w_F(\tau) \) and that \( w_{2c}(\tau) < w_{1c}(\tau) \). However, whether \( w_{1c}(\tau) \leq w_F(\tau) \) depends on the level of \( \gamma \). Indeed, if \( \gamma < \hat{\gamma} \) (as defined in the previous section), \( w_{1c}(\tau) < w_F(\tau) \), that is if the system is not very generous toward the non-working spouse, the number of single women supporting a specific tax rate
τ is higher than the one of individuals belonging to one-breadwinner couples will support the tax rate τ. On the contrary, if γ is high, we observe the reverse. These two cases are depicted on Figure 2.

We now turn to the determination of the equilibrium payroll tax rate under majority voting. The equilibrium tax rate is defined such that at least one half of the population prefers this tax rate (or a higher one) to any other lower tax rate. Since we can rank productivities, the voting equilibrium tax rate, τ∗ is then such that the number of individuals with higher wage (and thus who would prefer a lower tax level) represents exactly one half of the total population:

\[
(1 - \varphi) \left[ w^M (\tau^*) + \frac{w^F (\tau^*)}{\alpha} \right] + 2\varphi \mu w^{2c} (\tau^*) + 2\varphi (1 - \mu) w^{1c} (\tau^*) \geq 1 \quad (16)
\]

where a mass 1 of individuals corresponds to one half of the population. Solving the above equation (see appendix C), we obtain that

\[
\tau^* = \frac{1 - 3/\Omega (\alpha, \beta, \varphi, \mu, \gamma) \chi (\alpha, \beta, \varphi, \mu, \gamma)}{2 - 3/\Omega (\alpha, \beta, \varphi, \mu, \gamma) \chi (\alpha, \beta, \varphi, \mu, \gamma)} \quad (17)
\]

with \( \Omega (\alpha, \beta, \varphi, \mu, \gamma) = (1 - \varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi \left( \mu \sqrt{\frac{1 + \beta}{1 + \alpha^2}} + (1 - \mu) \sqrt{1 + \gamma \beta} \right) \)

We now illustrate this formula with a numerical example. We take as given \( \alpha = 0.8, \beta = 1.2 \) and \( \varphi = 0.6 \) and we focus on the incidence on the equilibrium tax rate of a variation in the number of two-breadwinner couples \( (\mu) \) and in the generosity of the pension system \( (\gamma) \). The results are reported in the Table 1.

<table>
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For any level of $\mu$, the equilibrium tax rate is increasing in $\gamma$, i.e. in the generosity toward one-breadwinner couples. In contrast, the variation of the tax rate with $\mu$, i.e. the number of two-breadwinner couples, is ambiguous. For instance, for very low levels of $\gamma (< 0.2)$, the tax rate is first decreasing in $\mu$ and then increasing in it, while for $\gamma \geq 0.3$, the tax rate is first increasing and then decreasing in $\mu$.

In the last years, we have observed a trend toward the individualization of pension rights (in our model this is equivalent to a decrease in $\gamma$) and an increase in the number of two-breadwinner couples (an increase in $\mu$). Since the effects of $\gamma$ and $\mu$ on the equilibrium tax rate may go in different directions, disentangling these effects would give us unclear conclusions. However, this situation of a lower $\gamma$ and a higher $\mu$ can be described by a move from the top right to the bottom left of the table; in this case, it is clear that the tax rate should decrease.

5 Conclusion

In this paper, we have focused on two individuals’ characteristics which are generally not taken into account in political economy models of social security: gender differences and marital status. To our knowledge, our analysis is the first to shed light on the importance of considering the couple as a distinct economic agent in order to explain the size of a pension system. As opposed to standard political economy models (which only consider single agents), we distinguish between single individuals, male or female, and couples. We also distinguish between one-breadwinner and two-breadwinner couples and account for the existence of derived pension rights.

We show that when there are only two productivity levels (one for men and one for women), single men and two-breadwinner couples are always against the pension system. On the contrary, one-breadwinner couples and single women may be in favor of it depending on the size of derived pension rights. This is related to the amount of redistribution these households get from the pension system. On the one hand, women benefit from the pension system because they have lower wages and higher longevity; on the other hand, one-breadwinner couples benefit from redistribution through derived pension rights. Thus, if derived pension rights are high, it is in favor of one-breadwinner couples but it is to the detriment of single women who can end up voting for a zero tax if these rights are not too high. We also extend our analysis by assuming a uniform distribution of productivity. We find that the equilibrium payroll tax should be decreasing when the system becomes less generous towards one-breadwinner couples. On the contrary, the relative
number of two-breadwinner couples is found to be ambiguous.

Clearly, our paper shows that the marital status and the composition of households influence the support for the pension system. Moreover, another main finding of our paper is that while two-breadwinner couples neutralize gender differences in longevity and productivity, one-breadwinner couples do not. If the pension system is generous towards the non-working spouses, these couples will push for high pension benefits. In that respect, we now observe two interesting evolutions in many countries: the progressive decline of the male sole breadwinner along with a non-working housewife and the individualization of pensions systems that impales less generosity towards non-working spouses. According to our model, these two evolutions should lead to a lower level of payroll taxation.

In our paper we make a number of simplifying assumptions: no liquidity constraints, labor supply invariant to the marital status, zero interest rate, actuarially fair annuity, no widowers, quadratic disutility of labor, quasi linear utility, uniform density of wages, assortative mating,... We do not think that the qualitative results would change if these assumptions were relaxed; at the same time, it is clear that the analytics would be more complicates.

Our model could still be extended in several directions. First, we do not model the incidence of adjusting the couples’ pension benefits for scale economies. Second, we only consider differences in longevity between men and women but we do not account for the empirical fact, that men have, on average, longer life expectancy when married than when singles. Regarding this second point, the intuition is that taking this feature into account would reinforce our results. In this case, the support for the pension system should increase as now, not only a married man would benefit from the advantage given to his wife but also he would receive a pension benefit for a longer period than if he had been single.
References


Appendix

A Preferred tax rate

We solve the following program:

\[
\max_{\tau \in [0,1]} V^i(\tau) = \frac{(1-\tau)^2 w_i^2}{2} - s_i^* + \pi_i u \left( \frac{s_i^*}{\pi_i} + \tau \frac{(1-\tau) E w^2}{\bar{\pi}} \right)
\]

Differentiating \(V^i(\tau)\) with respect to \(\tau\), we obtain

\[
\frac{\partial V^i(\tau)}{\partial \tau} = - (1-\tau) w_i^2 + \pi_i u'(d_i^*) \frac{E w^2}{\bar{\pi}} (1-2\tau)
\]

with \(u'(d_i^*) = 1\). Evaluating this expression at \(\tau = 0\), we find that any individual with \(w_i^2/E w^2 \geq \pi_i/\bar{\pi}\) always prefers a zero tax rate. For those with \(w_i^2/E w^2 < \pi_i/\bar{\pi}\), the solution is interior and the preferred tax rate is equal to (3).
B The one productivity model

Indirect utility functions of the four categories of population are

\[ V^F (\tau) = \frac{(1 - \tau)^2 \alpha^2 w^2}{2} - s^* + \pi \beta u \left( \frac{s^*}{\pi \beta} + p(\tau) \right) \]

\[ V^M (\tau) = \frac{(1 - \tau)^2 w^2}{2} - s^*_i + \pi u \left( \frac{s^*}{\pi} + p(\tau) \right) \]

\[ V^{c1} (\tau) = \frac{w^2 (1 - \tau)^2}{2} + (1 + \gamma \beta) \pi p(\tau) - (\beta + 1) \pi d^* + (\beta + 1) \pi u (d^*) \]

\[ V^{c2} (\tau) = \frac{(1 - \tau)^2}{2} (1 + \alpha^2) w^2 - (1 + \beta) \pi d^* + (1 + \beta) \pi p(\tau) + (1 + \beta) \pi u (d^*) \]

with \( p(\tau) \) defined by (8) and \( Ew^2 = w^2 \). Preferred tax rates are such that

\[
\frac{\partial V^F (\tau)}{\partial \tau} = -(1 - \tau) \alpha^2 w^2 + \pi \beta u' (d) \frac{dp(\tau)}{d\tau} \quad (17)
\]

\[
\frac{\partial V^M (\tau)}{\partial \tau} = -(1 - \tau) w^2 + \pi u' (d) \frac{dp(\tau)}{d\tau} \quad (18)
\]

\[
\frac{\partial V^{c1} (\tau)}{\partial \tau} = -(1 - \tau) w^2 + (1 + \gamma \beta) \pi \frac{dp(\tau)}{d\tau} \quad (19)
\]

\[
\frac{\partial V^{c2} (\tau)}{\partial \tau} = -(1 - \tau) (1 + \alpha^2) w^2 + (1 + \beta) \pi \frac{dp(\tau)}{d\tau} \quad (20)
\]

where \( u' (d^*) = 1 \) from first order conditions of the individual’s problem and where

\[
\frac{dp}{d\tau} = \frac{(1 - 2\tau)}{\pi} \chi \alpha, \beta, \varphi, \mu, \gamma w^2
\]

with \( \chi \alpha, \beta, \varphi, \mu, \gamma \) defined by (7). Evaluating \( \partial V^i (\tau) / \partial \tau \) at \( \tau = 0 \), we find that for male and two-breadwinner couples, \( \partial V^F (\tau) / \partial \tau < 0 \) and \( \partial V^{c2} (\tau) / \partial \tau < 0 \) so that their preferred tax rate is always zero (equation 9). For other groups, it is negative if

\[
\frac{\partial V^F (\tau)}{\partial \tau} \bigg|_{\tau=0} = \left[ \chi (\alpha, \beta, \varphi, \mu, \gamma) \beta - \alpha^2 \right] w^2 < 0 \quad \text{iff} \quad \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1}{\varphi (1 - \mu)} < \gamma
\]

\[
\frac{\partial V^{c1} (\tau)}{\partial \tau} \bigg|_{\tau=0} = \left[ \chi (\alpha, \beta, \varphi, \mu, \gamma) (1 + \gamma \beta) - 1 \right] w^2 < 0 \quad \text{iff} \quad \gamma > \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1 - \varphi + \varphi \mu}{1 + \alpha^2 (1 - \varphi + \varphi \mu) + \varphi (1 - \mu)}
\]
This defines threshold levels of $\gamma$ for women and one-breadwinner respectively:

$$\gamma_F = \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1}{\varphi (1 - \mu)}$$

$$\gamma_{c1} = \frac{\beta - \alpha^2}{\beta} \frac{1 - \varphi + \varphi \mu}{1 + \alpha^2 (1 - \varphi + \varphi \mu) + \varphi (1 - \mu)}$$

such that for $\gamma < \gamma_F$, the preferred tax rate of a single woman is always positive (resp. if $\gamma > \gamma_F$, her preferred tax rate is null) and for $\gamma > \gamma_{c1}$, the preferred tax rate of a one breadwinner couple is always positive (resp. if $\gamma < \gamma_{c1}$, their preferred tax rate is null). Thus, for $\gamma < \gamma_F$, the preferred tax rate level of women is strictly positive and solves the following equality

$$-(1 - \tau) \alpha^2 w^2 + \beta (1 - 2\tau) \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2 = 0$$

This yields (10). We use the same procedure for one-breadwinner couples when $\gamma_{c1} < \gamma$. The solution is interior and solves

$$-(1 - \tau) w^2 + (1 + \gamma \beta) (1 - 2\tau) \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2 = 0$$

which yields (11).

We also show that $\gamma_{c1} < \gamma_F$ as

$$\frac{1 - \varphi + \varphi \mu}{1 + \alpha^2 (1 - \varphi + \varphi \mu) + \varphi (1 - \mu)} < \frac{1}{\alpha^2 \varphi (1 - \mu)}$$

$$\iff -\alpha^2 [1 - \varphi (1 - \mu)] (1 - \varphi + \varphi \mu) < 1 + \varphi (1 - \mu)$$

which is always verified as the LHS is negative and the RHS is positive.

Finally we compare (10) and (11) and show that $\tau^*_F > \tau^*_{c1}$ if and only if

$$\gamma < \frac{\beta - \alpha^2}{\beta \alpha^2} \equiv \hat{\gamma}$$

It is straightforward to show that $\hat{\gamma} \in [\gamma_{c1}, \gamma_F]$. 

21
C Political equilibrium in a society with a distribution of productivity

C.1 Preferred tax rates

Substituting for
\[ \frac{dp(\tau)}{d\tau} = \frac{1}{3} \frac{(1 - 2\tau)}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma) \]

into (17), (18), (20) and (19), we obtain that

\[ \frac{\partial V^F(\tau)}{\partial \tau} \bigg|_{\tau=0} < 0 \text{ if } \frac{\alpha^2w^2}{1/3} > \beta\chi(\alpha, \beta, \varphi, \mu, \gamma) \Rightarrow \tau^*_F = 0 \]

On the contrary, if \( 3\alpha^2w^2 < \beta\chi(\alpha, \beta, \varphi, \mu, \gamma) \), the preferred tax rate is positive and such that \( \frac{\partial V^F(\tau^*_F)}{\partial \tau} = 0 \). In this case,

\[ \tau^*_F = \frac{\beta}{4} \chi(\alpha, \beta, \varphi, \mu, \gamma) - \frac{\alpha^2w^2}{2} \]

Using the same procedure for single men, we have that \( \frac{\partial V^M(\tau)}{\partial \tau} \bigg|_{\tau=0} < 0 \) if \( \frac{w^2}{1/3} > \chi(\alpha, \beta, \varphi, \mu, \gamma) \) so that in this case, \( \tau^*_M = 0 \), while for \( 3w^2 < \chi(\alpha, \beta, \varphi, \mu, \gamma) \), \( \tau^*_M \in [0, 1] \) and is equal to (13). For two-breadwinner couples, \( \frac{\partial V^{2c}(\tau)}{\partial \tau} \bigg|_{\tau=0} < 0 \) if \( 3w^2 > (1 + \beta) \chi(\alpha, \beta, \varphi, \mu, \gamma) / (1 + \alpha^2) \); otherwise, it is equal to (14). For one-breadwinner couples, \( \frac{\partial V^{1c}(\tau)}{\partial \tau} \bigg|_{\tau=0} < 0 \) if \( 3w^2 > (1 + \gamma\beta) \chi(\alpha, \beta, \varphi, \mu, \gamma) \); otherwise the solution is interior and equal to (15).
C.2 Equilibrium tax rate

Replacing for the expressions of $w^M(\tau^*)$, $w^F(\tau^*)$, $w^{2c}(\tau^*)$ and $w^{1c}(\tau^*)$ into (16) we get

\[
(1 - \varphi) \left[ w^M(\tau^*) + \frac{\sqrt{\beta}}{\alpha^2} w_M(\tau) \right] + 2\varphi \mu \sqrt{\frac{1 + \beta}{1 + \alpha^2}} w_M(\tau) + 2\varphi (1 - \mu) \sqrt{1 + \gamma \beta} w_M(\tau) = 1
\]

\[
w^M(\tau^*) \left[ (1 - \varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi \mu \sqrt{\frac{1 + \beta}{1 + \alpha^2}} + 2\varphi (1 - \mu) \sqrt{1 + \gamma \beta} \right] = 1
\]

\[
\sqrt{\frac{1 - 2\tau}{1 - \tau}} \frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} = \frac{1}{\Omega(\alpha, \beta, \varphi, \mu, \gamma)}
\]

where $\Omega(\alpha, \beta, \varphi, \mu, \gamma) = (1 - \varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi \left( \mu \sqrt{\frac{1 + \beta}{1 + \alpha^2}} + (1 - \mu) \sqrt{1 + \gamma \beta} \right)$

Rearranging terms, we obtain

\[
\tau^* = \frac{1 - 3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)}{2 - 3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)}
\]