Re-hypothecation of securities

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A public call to understand repo markets

An industry-led proposal to alleviate problems in the US government repurchase or repo market has brought a mixed response from analysts and brokers.

In a repo, a security is borrowed in return for a short term cash loan and such activity allows traders to sell Treasuries short without owning them. It also provides owners of Treasuries with funding when they lend them.

The repo market underpins trading activity in the US Treasury bond market. In a repo, a security is borrowed in return for a short term cash loan and such activity allows traders to sell Treasuries short without owning them. It also provides owners of Treasuries with funding when they lend them.

Lehman Brothers collapse

The repo market has been hit by a rise in so-called repo fails. A fail occurs when a security borrowed in exchange for a short-term cash loan is not returned on time. Failure to deliver borrowed securities reached a record $2.7bn last month, and the total remains at a still high level of $1.36bn.

The turmoil in repo has been compounded by the Federal funds rate effectively trading well below its target rate of 1 per cent in recent weeks. When rates are low, repo fails persist as the penalty for a fail is minimal.

This week, the Treasury Market Practices Group (TMPG), which comprises senior members of securities dealers, banks and investment firms and is sponsored by the Federal Reserve Bank of New York, set out a number of proposals, led by a call for implementing a greater penalty for repo fails. The TMPG wants the market to adopt a financial penalty for fails, calculated by a formula of 3 per cent minus the fed funds target rate. Such a penalty would be 2 per cent at this time and the crown wants the
• Repo markets attracted lot of attention in the recent credit crisis.

• However, repos have always been part of the tools at the central bankers disposal.

• The recent crisis revealed us how intricate funding, leverage and pricing are.
Recent call for regulation

- Lehman default
- Fail problem after Lehman
- Naked Short controversy
What is a repo?

- A repo trade = security sale + future repurchase of that amount at a predetermined date and price.
- Repo rate = future repurchase price – current price
- Is a repo a collateralized loan?

Why use repo markets?

- Borrower of cash (short in repo): To finance the purchase of a security.
- Borrower of the security (long in repo): To short a security through reverse repo.
The Repo Rate

- The difference between the future repurchase price and the current price corresponds to a level of interest rate which is called the *repo rate*. The repo rate $\sigma$ is a market level.

- General Collateral (GC) rate: the upper bound on the repo rate for a specific security.

- Specialness: the desirability of being in possession of the security pushes the repo rate below the GC rate (*Duffie 1996*)

  - The high cost of borrowing a security reflects its relative scarcity, in which case the security trades on special in the repo market.
Difference with a mortgage?  Re-hypothecation!

- **Mortgage**: A house pledged is sold only in default situation.

- **Repo**: Once a security has been pledged in a repo transaction it can be:
  - sold in the securities market
  - re-lend

- An equivalent security has to be returned at the end of the repo transaction (fungibility of securities is key) => *Fails*
Short sales constraints and the box: the missing inequality in financial theory

- The **box constraint**: non-negative **title balance** of the security.
  \[ e^i_{j,n} = e^i_{j,n^-} + y^i_{j,n} + z^i_{j,n} \geq 0 \]

- Getting **long** a security in the securities or repo markets both **increase** the amount of the security in the box.

- Getting **short** a security in the securities or repo markets both **decrease** the amount of the security in the box.

- The obligation to **reverse in** securities before shorting them re-establishes the upper hemi-continuity of the budget correspondence. The classical problem of the existence of equilibrium (Hart’s problem) is solved endogenously.
Shorting ≠ Issuing

- Issuers of securities (=> special powers in firms, and debt issuer)
- One needs to borrow a security to short it.
- Traditional Financial theory is linear on the financial variables
Leverage

The repo collateral multiplier

- Countably many transactions at each date.

- There is a security available, $C$, priced in the securities market at $q$. Let $c = qC$

- There is an exogenous haircut for repo operations, $0 < h < 1$, (the value of the loan is the haircutted value of the collateral)
The Repo Collateral Multiplier

<table>
<thead>
<tr>
<th>Moment 0</th>
<th>Cash Deposit</th>
<th>Repo Position</th>
<th>Security Position</th>
<th>Box Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. A</td>
<td>$c$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mr. B</td>
<td>0</td>
<td>0</td>
<td>$C$</td>
<td>$C$</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Step 1, Moment 1</th>
<th>Cash Deposit</th>
<th>Repo Pos.</th>
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<td>0</td>
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<thead>
<tr>
<th>Step 1, Moment 2</th>
<th>Cash Deposit</th>
<th>Repo Pos.</th>
<th>Security Pos.</th>
<th>Box Pos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. A</td>
<td>$hc$</td>
<td>$-C$</td>
<td>$C$</td>
<td>0</td>
</tr>
<tr>
<td>Mr. B</td>
<td>$(1 - h)c$</td>
<td>$C$</td>
<td>0</td>
<td>$C$</td>
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Step 2 starts and agents replicate Step 1. This is moment 3.

<table>
<thead>
<tr>
<th>Step 2, Moments 3</th>
<th>Cash Deposit</th>
<th>Repo Pos.</th>
<th>Security Pos.</th>
<th>Box Pos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. A</td>
<td>0</td>
<td>$-C$</td>
<td>$(1 + h)C$</td>
<td>$hC$</td>
</tr>
<tr>
<td>Mr. B</td>
<td>$c$</td>
<td>$C$</td>
<td>$-hC$</td>
<td>$(1 - h)C$</td>
</tr>
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Now Step 3 of the leverage build up starts.

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</thead>
<tbody>
<tr>
<td>Mr. A</td>
<td>0</td>
<td>-(1 + h)C</td>
<td>(1 + h + h^2)C</td>
<td>h^2C</td>
</tr>
<tr>
<td>Mr. B</td>
<td>c</td>
<td>(1 + h)C</td>
<td>-(h + h^2)C</td>
<td>(1 - h^2)C</td>
</tr>
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After the n iteration of repo operations followed by cash market operations:

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<tbody>
<tr>
<td>Mr. A</td>
<td>0</td>
<td>-(1 + h + ... + h^{n-1})C</td>
<td>(1 + h + ... + h^n)C</td>
<td>h^nC</td>
</tr>
<tr>
<td>Mr. B</td>
<td>c</td>
<td>(1 + h + ... + h^{n-1})C</td>
<td>-(h + ... + h^n)C</td>
<td>(1 - h^n)C</td>
</tr>
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</table>
In the limit the amount of the security in Mr. A and Mr. B’s box are 0 and C, respectively, which coincide with the initial positions in moment 0.

Net Mr. A has managed to leverage her cash $1/(1-h)$ times to build a security long position, without any uncertainty being resolved.
Modeling repo and security markets

General equilibrium model where delivery promises are fully honored.

- We consider an economy with \(J=\{1, \ldots, j, \ldots, J\}\) securities for trade in both the security and repo markets. Securities are real (pay in commodities).
- The set of agents is \(I=\{1, \ldots, i, \ldots, I\}\)
- The set of commodities is \(L=\{1, \ldots, l, \ldots, L\}\)
A model with repo rolls

Timing:

- Date 0: Agents trade in repo, securities and commodities.

- Date 1: Agents trade securities and commodities. (S states of nature)

- Date 2: Agents trade commodities. (S states of nature)
The constraints at date 0

- The budget constraint (BC.0):
  \[ p_0(x_0^i - \omega_0^i) + q_0y_0^i + \pi z^i \leq 0 \]
  where \( \pi_j = h_j q_{j0} \) and \( y_{j0}^i \) accounts for the gross trades on security \( j \) net of the security endowments, that is, \( y_{j0}^i = \varphi_{j0}^i - e_{j0}^i \)

- The box constraint (Box.0): non-negative balance of security title of ownership
  \[ y_{j0}^i + e_{j0}^i + z_j^i \geq 0, \quad \forall j \in J \]

- BC at date 1, state s,
  \[ p_s(x_s^i - \omega_s^i) + q_s y_s^i \leq p_s B_s (y_0^i + e_0^i) + r \pi z^i \]
  where \( r_j = 1 + \sigma_j \)

- BC at date 2, state s,
  \[ p_{s+}(x_{s+}^i - \omega_{s+}^i) \leq p_{s+} B_{s+} (y_s^i + y_0^i + e_0^i) \]
Pricing securities: liquidity

\[ q_{j0} = \sum_{\xi > 0} \frac{\lambda_0^i}{\lambda_0} p_\xi B_{j\xi} + \frac{\mu_{j0}^i}{\lambda_0} + \sum_{\xi > 0} \frac{\mu_{j\xi}^i}{\lambda_0} \]

New components
What do FOC tell us about the repo rate?

The FOC on repo trades is

$$\lambda_0 \pi_{j_0} = \sum_{\xi: t(\xi) = 1} \lambda_\xi r_{j_0} \pi_{j_0} + \mu_{j_0}$$

then

$$\sigma_{j_0} \equiv r_{j_0} - 1 = \frac{\lambda_0}{\sum_{\xi: t(\xi) = 1} \lambda_\xi} - \frac{\mu_{j_0}}{\pi_{j_0} \sum_{\xi: t(\xi) = 1} \lambda_\xi} - 1$$
Repo Specialness

\[ RS_j = RF - \sigma_j = \frac{\gamma^i_j}{h_j q_j} \]

where \( \gamma^i_j = \mu^i_j / \lambda^i_1 \) (the rent for borrowing the security)

- Repo specialness: when the repo rate of a specific security is below this GC rate.

- The necessity of an equilibrium approach: Repo rates are bounded by above (GC rate), but there is no obvious lower bound on such rate.

- If a security is on special there is an incentive for the owner of the specific security to lend it in the repo market. However, such opportunities are not scalable and are limited by the very scarcity of the security available at the date repo agreements are made.
Equilibrium

**Def:** An *equilibrium* for this economy consists on an allocation of *commodity bundles, net asset positions* and *repo positions* \((\bar{x}^i, \bar{y}^i, \bar{z}^i)\) together with the corresponding price vector \((\bar{p}, \bar{q}, \bar{r})\) such that:

- Commodities market clears: \(\sum_{i \in I} (\bar{x}^i - \bar{\omega}^i) = 0\), for all nodes \(\xi\).
- Securities market clears: \(\sum_{i \in I} \bar{y}^i = 0\) for all nodes \(\xi\).
- Repo market clears: \(\sum_i z^i_j = 0\), \(\forall j\).
Assumptions

- **A1**: \( \omega^i \gg 0, \forall i, \xi \)
  
  \( e^i_0 \gg 0, \forall i \)
  
  \( Du^i(x) \in \mathbb{R}^{(1+2S)\mathbb{L}}_{++} \quad [u^i]^{-1}(c) \) is closed in \( \mathbb{R}^{(1+2S)\mathbb{L}}_{++} \)
  
  \( h' D^2 u^i(x) h < 0, \forall h \neq 0 \) such that \( Du^i(x)h = 0 \)

- **A2**: \( B_j \in \mathbb{R}^{L}_{++}, \forall j. \)

- Under A1 and A2, if short sales are constrained, an equilibrium exists.
- Under A1 and A2, if the values of short sales and repo are constrained, an equilibrium exists.
Re-hypothecation

Definition 2: We call re-hypothecation rate (or re-hypo rate) $H$ the fraction of the amount of securities that can be sold or lent after being borrowed. We say that agents have to comply to no full re-hypothecation if $H < 1$.

- The modified box constraint: $\phi_{j0}^i + H z_{j}^{i+} - z_{j}^{i-} \geq 0, \ \forall j \in J$

Lemma 1: If $H_j < 1$ then (BoxH.0) implies that the values of security and repo positions are bounded, from above and from below.

Theorem 1: Let assumptions A1 and A2 hold. If there is no full re-hypothecation, then an equilibrium exists.
Proof of Lemma 1

Combining (BC.Hyp.0) with (BoxH.0) means that

\[ p_0 x_0^i + q_0 \phi_0^i + h q_0 z_0^{i+} \leq W_0^i + h q_0 (\phi_0^i + H z_0^{i+}) \quad \text{where } W_0^i \equiv p_0 \omega_0^i + q_0 e^i \]

Then

\[ q_j \phi_j^i \leq W_0^i / (1 - h_j) \quad \text{and} \quad q_j z_j^{i+} \leq W_0^i / h_j (1 - H_j) \]

As \( q_j (\phi_j^i + z_j^{i+}) \geq q_j (\phi_j^i + H z_j^{i+}) \), by (BoxH.0) we have

\[ q_j (\phi_j^i + z_j^{i-}) \leq W_0^i \left( \frac{1}{1 - h_j} + \frac{H_j}{h_j (1 - H_j)} \right) \]
How do re-hypo rates drive the amount of leverage?

\[ |z_j^i| \leq \frac{\sum_i e_{j0}^i}{1 - H_j} \]

Three institutional arrangements that end up limiting the amount of leverage:

- Segregated accounts
- Constrained dealers
- Repo pooling
Constrained dealers

- Non-dealer’s budget constraint:

\[ px^i + q(\phi^i + \theta^i - h\psi^i) \leq p_0\omega^i_0 + q_0e^i_0, \ i \in N \]  \hspace{1cm} (BC.nd)

- Dealer’s budget constraint:

\[ px^i + q(\phi^i + h\theta^i - \psi^i) \leq p_0\omega^i_0 + q_0e^i_0, \ i \in D \]  \hspace{1cm} (BC.d)

- We assume that:

(A3) Repos are only traded by non-dealers with dealers, whose security positions are bounded by regulation. Dealers collect haircut but do not pay haircut to non-dealers.
We have the following results:

**Lemma 2:** For non-dealers the values of security and repo positions are bounded, from above and from below.

**Theorem 2:** If $A1-A3$ hold, then an equilibrium exists.
Proof of Lemma 2

- The box constraint \( \phi_{j0}^i + \theta_j^i - \psi_j^i \geq 0 \) implies that \( \phi_{j0}^i + \theta_j^i - h_j \psi_j^i \geq 0 \)

- Then for \( i \in \mathbb{N} \) : \( q_{j0}(\phi_{j0}^i + \theta_j^i - h_j \psi_j^i) \leq p_0 \omega_0^i + q_0 e_0^i \)

- Again by the box constraint \( q_{j0}(\psi_j^i - \theta_j^i + \theta_j^i - h_j \psi_j^i) \leq p_0 \omega_0^i + q_0 e_0^i \)

- So,

\[
q_{j0} \psi_j^i \leq \frac{p_0 \omega_0^i + q_0 e_0^i}{(1 - h_j)}
\]

- Then for a dealer \( q_{j0} \theta_j^i \) is bounded

- By the dealer’s box constraint the values of dealers’ repo short positions are bounded and, therefore, the values of non-dealers’ repo long positions are bounded
It remains to bound non-dealers security positions

As
\[ q_{j0}(\phi^i_{j0} + h_j \theta^i_j - h_j \psi_j^i + (1 - h_j) \theta^i_j) \leq p_0 \omega_0^i + q_0 e_0^i \]

by the box constraint we get
\[ q_{j0}(1 - h_j)(\phi^i_{j0} + \theta^i_j) \leq p_0 \omega_0^i + q_0 e_0^i \]

So
\[ q_{j0} \phi^i_{j0} \leq \frac{p_0 \omega_0^i + q_0 e_0^i}{(1 - h_j)} \]

And \( \phi^{-i}_{j0} \) is also bounded in value as
\[ \sum_k \phi^k_{j0} = \sum_k \phi^k_{j0} + \sum_k e^k_{j0} \]
Fails Vs. Default

- A *fail* is the failure to return the security by the borrower of the security.
- A *default* is the failure to return cash by a borrower of funds.
- A default scenario is much more serious, triggering a full bankruptcy of the borrower of cash (the *short* in repo).
- In such situation, all the positions in the box of the short are sealed and given to the creditors.
Conclusions

- Securities market is not like derivatives market
- The box: scarcity of securities and specialness
- Re-hypothecation
- Issuance is not Shorting
- Leverage
- Equilibrium approach key to approach it all
The conclusion: