The impact of ambiguity on health prevention and insurance\textsuperscript{1}

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Abstract

In this paper, we analyze the choice of primary prevention made by individuals who bear a risk of being in bad health and an additive risk (of complications) that occurs after a disease has been diagnosed. By considering a two argument utility (depending on wealth and health), we show that the presence of a well-known (no ambiguity) additive risk of complications induces more investment in primary prevention by a risk-averse agent only if her preferences does not display some cross prudence in wealth ($u_{122} < 0$). If there is some ambiguity on the effective probability of complication, an increase in ambiguity aversion increases prevention if the agent is a correlation lover ($u_{12} > 0$). We also show that full (partial) insurance can be optimal even if insurance premia are loaded (fair). These results hold with and without prevention and the individuals attitudes toward correlation help explain the impact of ambiguity on the optimal individual decisions.

Key Words: health; utility; ambiguity; prevention; insurance.

JEL Codes: D81; I19.
1 Introduction

Causes of complications and death following the diagnosis of a disease are multiple and not always well known. In hospital, nosocomial infections and medication errors constitute major problems. Recently, medical researchers found that the incidence of nosocomial infections is associated with mortality in excess compared to mortality caused by the underlying diseases alone (see among others Gastmeier et al. (2007) and Grupper et al. (2007)). Besides, medication errors can lead to adverse drug events (Kopp et al. (2006), Sellier et al. (2009)) and individuals can react to these potential risks. Indeed individuals’ risk perception of nosocomial infection differs from the objective risk (Abate et al. (2008), Poujol et al. (2007)). A lack of information is generally suggested for explaining this gap and the existence of a risk of health complications or mortality can modify individuals’ behaviors in terms of health prevention.

In this paper, we propose to analyze individual choices of prevention made by individuals who bear, in addition to the risk of disease, an additive risk on the health status after disease has been diagnosed (nosocomial infection or medication errors for instance). Two types of prevention have been widely studied: (i) primary prevention which consists of helping avoid a given health care problem (such as education promoting the use of automobile passenger restraints, exercises, diet etc), and (ii) secondary prevention which corresponds to identifying and treating asymptomatic persons who have already developed risk factors or preclinical diseases but for whom the condition is not clinically apparent (for example screening tests for hypertension, breast or prostate cancer).

Our objective is to better understand the determinants of primary prevention (which corresponds to self-protection) when individuals are not well aware of the effect of the additive risk on health and its implications on insurance demand. A direct link between this setting and a concrete case can be easily made by focusing on the recent debate on the H1N1 virus. Indeed when considering two individuals in good health but one is more sensitive in case of disease, the question is to which extent this person should invest in more prevention in order to reduce the probability of being contaminated by the H1N1

\footnote{Vaccines are also part of primary prevention for they reduce the probability of disease to almost zero.}
virus, knowing that she has more chances to bear some complications (those persons are babies, older people, pregnant women, asthmatic individuals, etc).

The literature on microeconomics of prevention in the health sector proposes several papers on the individual trade-offs between prevention and care (Eeckhoudt, Godfroid and Marchand (1998)) and between prevention and insurance (Zweifel et al. (2009)). In these models, individuals are supposed to have a perfect knowledge of the characteristics of their risk and of the effectiveness of prevention. Nevertheless perfect information about complications is not the rule. Instead, one observes often some hesitations of the medical staff and, a fortiori of the individuals, when they have to evaluate the chances that complications might occur in case of a given disease. Hence the probability of having to face an additive risk is not always well known and individuals have to take decisions in an ambiguous environment: risk aversion and ambiguity aversion are, finally, two concepts that play an important role in this decision process as recently showed, in a one argument-utility model, by Alary, Gollier and Treich (2010).

Since Ellsberg (1961), a large experimental literature has confirmed that individuals are averse to ambiguity, while a large theoretical literature has developed models to accommodate this behavior (Gilboa and Schmeidler (1989), Gajdos, Hayashi, Tallon and Vergnaud (2008) and Klibanoff et al. (2005) among others). In this paper, we consider the Klibanoff, Marinacci, Mukerji (2005) model in order to distinguish risk aversion from aversion to ambiguity.

As did Dardanoni and Wagstaff (1990), we consider a two argument utility (depending on wealth and health). We show that the presence of a well-known (no ambiguity) additive risk induces more investment in primary prevention by a risk-averse individual only if her preferences do not display some cross prudence on wealth (i.e. if \( u_{122}(\cdot,\cdot) \leq 0 \)). Hence we obtain a result close to the one obtained by Eeckhoudt and Gollier (2005) in an expected utility model with well-known risks or to those proposed by Courbage and Rey (2006) when individuals present some fear of sickness: prudence and prevention can be opponents. The cross prudence concept, with the aversion to correlation, is developed in Eeckhoudt, Rey and Schlesinger (2007). It is notably showed how a variation in one argument of the utility function can either mitigate or aggravate the impact of a variation.
in the other argument of the utility function. While cross prudence of wealth deals with the impact of a variation of health on the variation of the marginal utility of wealth, aversion to correlation implies that the individual dislikes a decrease of her wealth when her health is already deteriorated.

Now, if there is some ambiguity on the effective probability of complication in case of illness, then an increase of the ambiguity aversion increases prevention only if an improvement of the health status increases the marginal utility of monetary wealth: This means that the individual is correlation loving. A higher wealth is no longer valuable if her health is deteriorated and, consequently, expenses in prevention can increase. We still show how some public prevention and private prevention are either substitutes or complements, depending on the type of the population at stake. Still here, the attitude of the agents toward a correlation between the variations in wealth and in health helps interpret the results.

In the second part of the paper, we obtain some original results related to insurance. In particular, we show that full insurance can be optimal even if insurance premia are loaded. More precisely, individuals that are averse to correlation dislike a decrease in their wealth when their health is deteriorated: thus they prefer full insurance to partial coverage even if it is (not too) costly, namely when the loading factor of the insurance premium is strictly positive. On the contrary, with fair premia, an individual will not always request full coverage when she is ambiguity averse but correlation loving. These results hold with and without prevention. They highlight the fact that some individuals are willing to be over-insured in the presence of ambiguity, which is never the case in standard expected utility models with one, well known insurable risk (Arrow (1963), Raviv (1979), Gollier and Schlesinger (1996), Spaeter and Roger (1997)). Our results are much closer to those obtained by Jelleva (2000) with a non additive model or by Doherty and Eeckhoudt (1995) within a Yaari framework.

The paper is organized as follows. Section 2 deals with the model, while Section 3 presents the impact of ambiguity and risk aversion on the optimal level of prevention without insurance. In Section 4, we assume that the individuals can also buy insurance. Section 5 concludes the paper.
2 The Model

We consider a static model in which an individual derives utility from consumption and from the quality of her health status.

This individual agent faces a health risk. With probability \( p \), she gets ill at the end of the period and her health status is \( H \) (if considering only this risk). With probability \( (1 - p) \), she remains in good health, \( \overline{H} \). She bears also an additive risk \( \tilde{e} \), which represents potential complications in case of illness which are due to an exogenous factor. For the sake of simplicity, we consider two possible events: the complication appears with probability \( q \) or does not appear with probability \( 1 - q \). Formally, this additive risk is defined by the random variable \( \tilde{e} = (0, 1 - q; \epsilon, q) \), \( \epsilon > 0 \). As a direct consequence, the health status in case of illness becomes \( H - \tilde{e} \), and it is random, while it remains \( \overline{H} \) in the good state. Both random variables \( \tilde{e} \) and \( \overline{H} \) are independently distributed.

The ambiguity concerns the probability \( q \) of complications, which is perfectly known neither by the individual nor by the insurer considered in Section 4. We denote as \( F(q) \), the distribution of the random probability \( \tilde{q} \) on \([0, 1]\) with \( F(0) = 0 \) and \( F(1) = 1 \).

At the beginning of the period, the individual receives a revenue \( w \), and chooses the amount \( h \) she will invest in primary prevention. The cost of one unit of prevention is normalized to one, and \( h \) units of prevention allow the agent to improve the probability \( p \) of a good state of nature according to a technology represented by \( p(h) \), with: \( p'(h) < 0 \) and \( p''(h) > 0 \). Thus, the certain net wealth of the agent is \( w - h \), while her final health risk is \( \tilde{H} = (\overline{H}, H, H - \epsilon; 1 - p, p, (1 - \tilde{q}), p, \tilde{q}) \). Her preferences over wealth and health are represented by the utility function \( u(w - h, \tilde{H}) \).

For a given distribution \( q \), the agent computes the expected utility:

\[
U(h, q) = p(h)[qu(w - h, H - \epsilon) + (1 - q)u(w - h, H)] + (1 - p(h))u(w - h, \overline{H})
\]

(1)

Function \( u(\ldots) \) depends on final wealth and on health. We suppose that in case of death, \( H = 0 \), utility is equal to zero. It satisfies \( u_1 > 0 \), \( u_2 > 0 \), \( u_{11} < 0 \) and \( u_{22} < 0 \).\(^3\)

\(^2\)Another situation, which is out of the scope of this paper, could be to consider that the agent is a hospital, better informed than the insurer about the risk of complications.

\(^3\)Index \( i \) denotes a partial derivative of the utility function with respect to argument \( i \) (\( i = 1 \) or 2).
The individual dislikes a reduction in any attribute, ceteris paribus. And, as it is usually assumed (see, among other papers, Courbage and Rey, 2006), the individuals are risk-averse toward each separate risk: the wealth risk and the health risk. They dislike any additive mean-zero risk on one or the other argument, ceteris paribus.

Following Klibanoff, Marinacci, Mukerji (2005), ambiguity aversion is characterized by a concave function $T$, defined over $U(.,.)$. With risk aversion being represented by the concavity of function $u$, the individual’s expected welfare is measured by $V(h)$, defined as follows:

$$V(h) = \int_0^1 T[U(h, q)]dF(q) \quad (2)$$

with $T'(.) > 0$ and $T''(.) \leq 0$. Just as the concavity of $U$ represents the aversion of the individual toward risk, the concavity of function $T$ fits with ambiguity-aversion.

Individuals are willing to pay a positive amount in order to eliminate ambiguity if it is possible (that is in order to know with certainty the effective value of $q$).

The agent’s program writes finally: $\max_h V(h)$.

**Lemma 1** The optimal conditions for an interior solution are:

$$\int_0^1 T'[U(h, q)].\frac{\partial U(h, q)}{\partial h}dF(q) = 0 \quad (3)$$

and

$$\int_0^1 \left[ T''[U(h, q)].\left(\frac{\partial U(h, q)}{\partial h}\right)^2 + T'[U(h, q)].\frac{\partial^2 U(h, q)}{\partial h^2}\right]dF(q) < 0 \quad (4)$$

with

$$\frac{\partial U(h, q)}{\partial h} = p'(h)[qu(w - h, H - \epsilon) + (1 - q)u(w - h, H)]$$

$$- w(w - h, H)]$$

$$- p(h)[qu_1(w - h, H - \epsilon) + (1 - q)u_1(w - h, H)]$$

$$- (1 - p(h))u_1(w - h, H) \quad (5)$$

The optimal conditions are very close to those obtained without ambiguity. The first term corresponds to the attitude toward ambiguity. The second term which is similar to the standard one in prevention analysis is the difference between the expected utility
benefit of reducing the probability of illness and the expected marginal value of final wealth.

3 Ambiguity and Risk Aversion

In this section we analyze the impact of risk aversion and ambiguity on the optimal investment in prevention made by the agent.

3.1 The impact of the presence of an additive risk

First let us consider that there is no ambiguity\(^4\): The probability of complications is given and known. We denote it \(\tilde{q}\). Then function \(T(.\)) is the identity function and the optimal level of prevention \(\tilde{h}\) satisfies:

\[
\tilde{h} = \frac{\partial U(h, \tilde{q})}{\partial h} \bigg|_{h=\tilde{h}} = 0
\] (6)

**Proposition 1** For a given, unique, and strictly positive probability \(\tilde{q}\) of the additive risk, a risk averse individual will always invest more in prevention than without the additive risk if \(u_{122}(.,.) \leq 0\). On the contrary, if the agent’s preferences exhibit some sufficient cross prudence in wealth \((u_{122}(.,.) > 0)\), prevention decreases.

**Proof.** See Appendix. ■

When utility is additively separable in wealth and health and when the preferences exhibit some cross imprudence in wealth \((u_{122} \leq 0)\) as defined by Eeckhoudt, Rey and Schlesinger (2007), a risk-averse agent will always increase her optimal level of prevention in the presence of an additive risk. Indeed, cross prudence in wealth means that a higher wealth helps temper the detrimental effects of accepting the additive health risk. Thus, if the preferences of the individual display some cross imprudence in wealth, the effect of the additive risk on the health status is aggravated by a higher wealth. And a higher level of financial investment in prevention just lessens the wealth level.

Besides, if some cross prudence in wealth is observed \((u_{122} > 0)\), it is not possible to conclude without focusing on the effective degree of cross prudence. Indeed, if considering

\(^4\)Or, equivalently, that the agents are ambiguity neutral.
the proof of Proposition 1, it is possible to show that prevention still increases with low levels of the cross prudence degree, while it diminishes for agents largely (cross) prudent in wealth. This result is due to the fact that we do not consider a mean-zero additive risk on the health status as it is done by Eechoudt, Rey and Schlesinger (2007) but a pure risk of health deterioration. Thus by additive the risk \( \bar{\varepsilon} \), the health status in the bad state of nature is always deteriorated compared to a situation in which no additive risk exists. Thus, more than just some positive prudence is needed in order to reverse the result compared to the case with \( u_{122} \leq 0 \). Finally, by focusing on sufficiently high coefficients of cross prudence, we are able to obtain a result close to the one of Eeckhoudt and Gollier (2005). Cross prudence leads to less prevention: If \( h \) decreases, then the final wealth increases, mitigating the negative effect due to the presence of the additive risk on the health status.

In what follows, prudence still explains some differences between the respective individual strategies of two different agents. In particular, we obtain Courbage and Rey (2006) result, thanks to their definition of fear of sickness but in a model with an additive risk. The agent’s preferences display some fear of sickness if she dislikes a decrease

**Proposition 2** Let us consider two agents, 1 and 2. For a given, unique, strictly positive and lower than one probability \( \hat{q} \) of the additive risk, if Agent 1 has more fear of sickness but is less cross prudent in health than Agent 2, then Agent 1 will invest more in prevention than Agent 2.

**Proof.** See Appendix. ■

This result highlights one more time the fact that higher prudence can lead to less prevention as in Eeckhoudt and Gollier (2005). The more prudent individual, the higher

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5 "Fear of sickness measures the 'degree of future pain' induced by the occurrence of the illness" (Courbage and Rey, 2006,p 1324). Formally, if Agent 1 has more fear of sickness than Agent 2, then there exists a function \( K(\cdot) \) with \( K'(\cdot) > 1 \) such that:

\[
\begin{align*}
u^2(\cdot, \overline{H}) &= \nu^1(\cdot, \overline{H}) \\
u^2(\cdot, H) &= K(\nu^1(\cdot, H)) \quad \forall H < \overline{H}.
\end{align*}
\]

It is worth noticing that fear of sickness implies risk aversion.
the final wealth she needs to mitigate the negative effect of the health risk: this implies lower expenses for prevention.

The results of Propositions 1 and 2 serve as benchmarks for the course of the paper, where ambiguity (subsections 3.2. and 3.2.) and insurance (Section 4) are introduced.

3.2 Ambiguity aversion

To analyze the attitude toward ambiguity, we consider a change in the transformation function $T$. We have to mobilize the concept of correlation aversion analyzed by Eeckhoudt, Rey and Schlesinger (2007). In a two-argument utility model, wealth and health for instance, a higher level of wealth mitigates the detrimental effect of a reduction in the health status for a correlation averse agent. In terms of utility, correlation aversion is formalized by $u_{12} < 0$.

**Proposition 3** The optimal prevention increases with ambiguity aversion if individuals are correlation loving, that is if $u_{12} \geq 0$.

**Proof.** Let Agent 1 with $T_1$ be less ambiguity-averse than Agent 2 with $T_2$. Similarly to the risk-aversion analysis, there exists an increasing and concave function $k(.)$ such that $T_2 = k(T_1)$. As utility function $U(h, q)$ is a decreasing function of $q$, we have for any $h$ and for all $q$ in $[0, 1]$, $U(h, q) < U(h, 0)$. Then, for all $q$ and all increasing function $T$, we have $k'(T[U(h, q)]) > k'(T[U(h, 0)])$. This implies that

$$\int_0^1 T_2[U(h^1, q)] \frac{\partial U(h^1, q)}{\partial h} dF(q) = \int_0^1 k'(T_1)T_1'[U(h^1, q)] \frac{\partial U(h^1, q)}{\partial h} dF(q)$$

$$> k'(T_1[U(h^1, 0)]) \frac{\partial U(h^1, 0)}{\partial h} \int_0^1 T_1'[U(h^1, q)] dF(q)$$

if $u_{12}$ is positive.

This last term is also positive if $u_{12}$ is positive (see 5). Finally, the first order condition of Agent 2 evaluated at $h = h^1$ is positive so that $h^1 < h^2$. ■

As mentioned by Eeckhoudt, Rey and Schlesinger (2007) correlation aversion may not be the rule in terms of realistic preferences. Indeed, a deterioration of health can be so painful for an individual that no wealth could mitigate it. Thus $u_{12}$ can also be positive,
representing the preferences of correlation loving agents. It has been empirically showed by Viscusi and Evans [1990] and Sloan et al. [1998], that \( u_{12} > 0 \) rather concerns severe injuries. For minor ones, Evans and Viscusi [1991] find that \( u_{12} \) can be negative.

Technically, \( u_{12} \geq 0 \) being a sufficient, but not necessary, condition for our result to hold, it can also apply to not too negative values of \( u_{12} \).

From a positive point of view, the result of Proposition 3 is in line with what is sometimes observed for large injuries. Money becomes less valuable if the individual is not healthy enough to benefit from it. Thus, an increase in prevention is less costly in terms of wealth loss for individuals more ambiguity averse when they are also correlation lovers. Higher prevention does not help mitigate the negative effect due to the presence of the additive health risk here, but rather the effect of ambiguity on its effective probability of realization.

### 3.3 Increasing Risk

One crucial issue concerns the individuals’ responses to a change in the risk of complications. Indeed, public authorities or medical services in hospitals can control the probability of bad state of nature (facing some complications after a disease has been diagnosed) by adopting some preventive measures or, on the contrary, by cutting into some types of expenses. Since Rothschild and Stiglitz (1970), different measures (or definition) of an increase in risk have appeared in the literature. The most intuitive one in our case is certainly the basic first order stochastic dominance.

Indeed a decrease of the risk of complications in the sense of the first order stochastic dominance means that the probability \( q \) of facing some complications decreases, whatever \( q \). Formally, the distribution \( F(q) \) changes to a distribution \( G(q) \) with \( G(q) \geq F(q) \) for any \( q \) and with, at least, one strict inequality. In other non technical words, it is always more likely to face some complications with distribution \( F \) than with distribution \( G \).

**Proposition 4** Optimal prevention increases following an increase in the risk of complications in the sense of the first-order stochastic dominance, if \( u_{12} \geq 0 \) and ambiguity aversion is not too high.
Proof. See Appendix. ■

Private prevention and public prevention can be viewed as either substitutes or complements according to the level of ambiguity aversion. Indeed, let us consider the symmetric situation of Proposition 4 and suppose that medical services can mitigate the risk of complications (in the sense of the first order stochastic dominance). Then some agents may invest less in prevention. This is the case, in particular, for agents not too ambiguity-averse and presenting some correlation love: if the total risk of illness is mitigated (the situation is improved), they also improve their wealth status by lessening the preventive expenses. But if individuals are highly ambiguity averse, they may increase their level of prevention, because their private activity of risk mitigation becomes more efficient, while public prevention is not sufficient for them.

4 Prevention and Insurance

Let us assume in this section that in case of illness, there exist some treatments which allow the agent to improve her health status. We also assume that private insurance is available. Treatments and their cost depend on the type of illness. It is fair to assume that the treatment cost if complications are observed, denoted \( c(\epsilon) \), is larger than the one without complications, \( c(0) \). For simplicity’s sake and without loss of generality, we also assume that the health status after treatment, denoted \( H^t \), holds for bad states both with and without complications. Nevertheless it cannot be larger than the one relative to good health. Formally we have \( H < H^t < \bar{H} \).

The insurance market is competitive: insurance firms act as risk neutral and ambiguity neutral agents. The compensation, \( I \), paid by the insurance company to the insured agent when she is ill, depends on the treatment cost: \( I(\bar{\epsilon}) = \alpha c(\bar{\epsilon}) \) where \( \alpha \) is the

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6Thus we do not make any difference with respect to duration. This is done by Bleichrodt, Crainich and Eeckhoudt (2003) who analyze the impact of comorbidities on medical decisions. They consider a two argument utility, depending on the quality of life and on duration. But wealth is not an explicit variable of their model and ambiguity is not considered.

7No extra premium can be confiscated from the insured agents.
copayment\textsuperscript{8}. The insurance premium, $\Pi$, is defined on the basis of the expected value of indemnity,

$$
\Pi = \alpha(1+\lambda)E_qE_c(\epsilon),
$$
where $\lambda$ is the loading factor covering, in particular, administrative costs of the insurer. Precisely we have:

$$
\Pi = \alpha(1+\lambda)p(h) \int_0^1 (qc(\epsilon) + (1-q)c(0))dF(q)
$$
where $h$ is the level of primary prevention.

\textbf{4.1 Insurance without prevention}

First, let us analyze the demand for insurance without the possibility of investing in prevention (Subsection 4.2. deals with both prevention and insurance). Here, we have $p(h) = p$ and the insurance premium writes

$$
\Pi = \alpha(1+\lambda)p \int_0^1 (qc(\epsilon) + (1-q)c(0))dF(q).
$$
(7)

Each agent chooses the level of coinsurance, $\alpha$, which maximizes her utility. The optimal level of coinsurance satisfies

$$
\alpha^* \arg \max V(\alpha) = \int_0^1 T[U(\alpha, q)]dF(q)
$$
(8)

with

$$
U(\alpha, q) = p[qu(w-\Pi+I(\epsilon)-c(\epsilon), H^1)+(1-q)u(w-\Pi+I(0)-c(0), H^1)]+(1-p)u(w-\Pi, H)
$$
and $\Pi$ defined by (7).

\textbf{Proposition 5} If $u_{12} \leq 0$ and $\lambda \leq \lambda_0$ with $\lambda_0 > 0$, full insurance coverage is optimal. If $u_{12} > 0$, full insurance is never optimal whatever the value of $\lambda$.

\textsuperscript{8}We assume here that only pure coinsurance contracts are proposed to the agents. Even if insurance economics shows that deductibles are usually Pareto improving compared to coinsurance (Arrow, 1963; Raviv, 1979; Gollier and Schlesinger, 1996; Spaeter and Roger, 1997) our assumption remains fair. First, working with a two state model as we do induces that a deductible contract and a coinsurance contract are equivalent (see Doherty and Eeckhoudt, 1995, for instance). Second, from a more positive point of view, coinsurance contracts are still observed in private health insurance (sometimes associated with deductibles).
Proof. See Appendix. ■

This results hold whatever the attitude toward ambiguity of the agent and they differ from some standard results in insurance economics. First, if $u_{12} \leq 0$, full insurance remains the optimal contract even if insurance is costly ($\lambda > 0$) and this holds even for ambiguity neutral agents. It should be recalled that $u_{12} \leq 0$ concerns an agent who presents some correlation aversion. Hence a higher level of wealth can mitigate the negative effect she bears in case of complications. Thus she prefers full insurance for her financial wealth even if it is costly for her. This result contrasts with those obtained in a well-known risk setting (Arrow (1963), Raviv (1979), Gollier and Schlesinger (1996), Spaeter and Roger (1997), ...). Besides, such a result is to be brought closer to the results obtained by Doherty and Eeckhoudt (1995) with a Yaari model or by Jeleva (2000) in a Choquet capacity context\(^9\). They also obtain the optimality of full insurance in some cases with loaded insurance premia, questioning once more the universality of the pareto-optimality of partial insurance when insurance is costly.

The second result of Proposition 5 is that $u_{12} > 0$ always leads to the optimality of partial insurance even without any loading factor of the insurance premium. And $u_{12} > 0$ means that the individual is correlation loving: the existence of the additive risk on the health status does not give her sufficient incentives to obtain the highest possible wealth level in case of complications. She prefers to retain part of her financial risk. Thus the question of the superiority of full insurance over partial insurance in an environment without any fraud considerations and any administrative costs is still to be discussed.

Now, when considering more practical features, it seems fair to assume that administrative costs are rather large in the health sector and, in particular at least as high as in non-life insurance (they account for 30% of the premium in car insurance for instance). Thus from the above results, partial insurance should be the rule. Nevertheless, Proposi-

\(^9\)In this latter model, the optimality of full insurance when the premium is loaded is obtained when the agent is more pessimistic than the insurer about her probability of loss. In our setting, ambiguity holds for both agents and the insurer is less ambiguity averse than the insured individual (actually, he is ambiguity neutral).
Proposition 6  Ambiguity aversion increases the level of insurance.

Proof. See Appendix. ■

As for risk aversion, the effect of ambiguity aversion on optimal insurance is unambiguous. Moreover, ambiguity aversion reinforces risk aversion: The insurance demand increases.

4.2 Increasing Risk

As for prevention in Subsection 3.3, we want to analyze the effect of an increase of the risk of complications on the optimal demand of insurance. Still here, we consider first order stochastic dominance changes of the distribution $F(q)$.

Proposition 7  The insurance demand increases with an increase in "risk of complications", in the sense of the first-order stochastic dominance, if ambiguity aversion is not too high.

Proof. It is similar to the proof of Proposition 4. ■

If public prevention makes it possible to mitigate the risk of complications, it can induce individuals to decrease or increase their level of insurance demand. Indeed, let us suppose one can diminish the probabilities of complication. This "good" change in distribution can give the agents some incentives to diminish their insurance demand. But if individuals are sufficiently ambiguity averse, insurance demand can increase. As a direct consequence of Proposition 7, public prevention and insurance can be complements for high ambiguity averse agents and substitutes for others. Hence a public policy the aim of which is to contribute to the mitigation of the risk of complications (at hospital for instance) will have different effects on the demand of insurance, depending on the population that is at stake.

Finally, insurance demand and preventive investment behave similarly when considered separately. Both an increase in the ambiguity aversion of the agent and an
increase in the risk of complications may increase insurance demand. It was also the case for prevention when considered alone in the preceding section. Do these results still hold when insurance and private prevention are considered simultaneously by the agent as two activities that can mitigate the health risk?

4.3 Insurance with prevention

Now, let us assume that the agents can buy insurance and, simultaneously, invest in prevention. Each agent has to choose the level of coinsurance, $\alpha$, and the level of prevention, $h$, which maximize her utility. Prevention is observable by the insurer and it is taken into account when the premium is computed. For a given distribution $q$ of $\bar{\varepsilon}$, the agent computes now the expected utility:

$$U(\alpha, h, q) = p(h)[qu(w - \Pi - (1 - \alpha)c(\varepsilon) - h, H^\dagger) + (1 - q)u(w - \Pi - (1 - \alpha)c(0) - h, H^\dagger)]$$

$$+ (1 - p(h))u(w - \Pi - h, \Pi)$$  \hspace{1cm} (9)

And her maximisation program writes:

$$\max_{\alpha, h} V(\alpha, h) = \int_0^1 T[U(\alpha, h, q)]dF(q)$$ \hspace{1cm} (10)

s.t.

$$\Pi = \alpha(1 + \lambda)p(h) \int_0^1 (qc(\varepsilon) + (1 - q)c(0))dF(q)$$  \hspace{1cm} (11)

**Proposition 8** When the agent can simultaneously buy proportional insurance and invest in prevention, full insurance coverage is optimal if $u_{12} \leq 0$ and $\lambda \leq \lambda_1$ with $\lambda_1 > 0$. If $u_{12} > 0$, full insurance is never optimal.

**Proof.** See Appendix.  \hspace{1cm} ■

The non systematic optimality of partial insurance when insurance premia are loaded still holds if the agent can make some prevention. Still here, prudence is not necessary to obtain the result but correlation attitude.
5 Conclusion

In this paper we have investigated the investment in prevention and/or in insurance by an individual who faces two risks: a first risk of being in bad health and a second one, which takes place only in the bad health state and which worsens it. While the distribution of the first health risk is well known by all the agents, the distribution of the risk of complications is not perfectly known and ambiguity holds. This last assumption illustrates well many realistic situations (nosocomial infections at hospital, relative ignorance about one’s resistance in case of illness, ...) although it has seldom been considered in health economics.

Attitude toward risk and toward ambiguity are defined from a two argument utility function. It depends on the health status and on wealth. Thus we were able to use the really meaningful concept of correlation aversion developed in Eeckhoudt, Rey and Schlesinger (2006). We also refer to cross prudence as done in Eeckhoudt, Rey and Schlesinger (2007) and in Courbage and Rey (2006).

First we showed that in the absence of ambiguity a risk-averse individual invests more in primary prevention only if her preferences do not display some cross prudence on wealth (i.e. if $u_{122} (.,.) \leq 0$). Our result is close to the one obtained by Eeckhoudt and Gollier (2005) in an expected utility model with well-known risks or to those proposed by Courbage and Rey (2006) when individuals present some fear of sickness: prudence and prevention can be opponents.

If there is some ambiguity on the effective probability of complication in case of illness, then an increase of the ambiguity aversion increases prevention only if an improvement of the health status increases the marginal utility of monetary wealth. In that case the individual is correlation loving. A higher wealth is no longer valuable if her health is deteriorated and, consequently, expenses in prevention can increase. By considering changes in the risk of complications in the sense of first order stochastic dominance, still we showed how some public prevention and private prevention are either substitutes or complements, depending on the type of the population at stake. Precisely, a more ambiguity averse agent will more often increase her level of prevention if the risk of
complications is mitigated. On the contrary, a not too ambiguity averse individual will prefer to consider public prevention and private prevention as complements. From a decision-making perspective, those results suggest that a public policy the aim of which would be to mitigate some kinds of additive health risks, at hospital for instance or within a specific category of individuals, should give room to the risk and ambiguity attitudes of the concerned individuals.

In the second part of the paper, we considered also insurance of medical treatments. We showed that full insurance can be optimal even if insurance premia are loaded. Individuals that are averse to correlation dislike a decrease in their wealth when their health is deteriorated, thus preferring full insurance to partial coverage even if it is (not too) costly. On the contrary, with fair premia, an individual will not always demand full coverage when she is ambiguity averse but correlation loving. These results hold with and without prevention and they highlight the fact that some individuals are willing to be over-insured in the presence of ambiguity, which is never the case in standard expected utility models with one well known insurable risk (Arrow (1963), Raviv (1979), Gollier and Schlesinger (1996), Spaeter and Roger (1997)). Our results are much closer to those obtained by Jeleva (2000) with a non additive model or by Doherty and Eeckhoudt (1995) within a Yaari framework.

**APPENDIX**

**Proof of Proposition 1**

(1) and (6) yield:

\[
0 = p'(\hat{h})[\hat{q}u(w - \hat{h}, H - \epsilon) + (1 - \hat{q})u(w - \hat{h}, H) - u(w - \hat{h}, \bar{H})] \\
- p(\hat{h})[\hat{q}u_1(w - \hat{h}, H - \epsilon) + (1 - \hat{q})u_1(w - \hat{h}, H)] - (1 - p(\hat{h}))u_1(w - \hat{h}, \bar{H})
\]

Let us suppose now that the additive risk is a degenerate random variable that takes the certain value $\hat{q}\epsilon$. In case of illness, the health status is equal to $H - \hat{q}\epsilon$. The optimal level of prevention $h_0$ satisfies in such a situation:

\[
0 = p'(h_0)[u(w - h_0, H - \hat{q}\epsilon) - u(w - h_0, \bar{H})] \\
- p(h_0)u_1(w - h_0, H - \hat{q}\epsilon) - (1 - p(h_0))u_1(w - h_0, \bar{H})
\]
Consequently, an individual will invest more in prevention in the presence of an additive risk if and only if

\[ 0 > p(h_0)[u(w - h_0, H - \tilde{\epsilon}) - (\tilde{q}u(w - h_0, H - \epsilon) + (1 - \tilde{q})u_1(w - h_0, H))] \]

\[ -p(h_0)[u_1(w - h_0, H - \tilde{\epsilon}) - (\tilde{q}u_1(w - h_0, H - \epsilon) + (1 - \tilde{q})u_1(w - h_0, H))] \]

which is true if the individual is risk averse \((u_{22} < 0)\) and \(u_{122} \leq 0\). If \(u_{122} > 0\) it is not possible to conclude about the sign of the concerned expression. Proposition 1 is demonstrated.

\[ \diamond \]

**Proof of Proposition 2.**

Without ambiguity on the adding risk and from (2) and (1), the first order condition for Agent \(i\) is:

\[ h^i/V^i_k(h^i) = \frac{\partial U^i(h, q)}{\partial h} |_{h=h^i} = 0 \]

\[ \iff -p'(h^i)v^i(h^i) - Eu^i_1(w - h^i, \tilde{H} - \tilde{\epsilon}) = 0 \]

with \(v^i(h^i) = u^i(w - h^i, \overline{H}) - [\tilde{q}u^i(w - h^i, H - \epsilon) + (1 - \tilde{q})u^i(w - h^i, H)]\) and

\[ Eu^i_1(w - h^i, \tilde{H} - \tilde{\epsilon}) = p(h^i)[\tilde{q}u_1^i(w - h^i, H - \epsilon) + (1 - \tilde{q})u_1^i(w - h^i, H)] + (1 - p(h^i))u_1^i(w - h^i, H), \]

the expected marginal utility of wealth.

An agent is cross prudent in health iff \(u_{122}(., .) > 0\). If Agent 1 is less cross prudent than Agent 2 in health, then \(u_1^i\) is less convex than \(u_2^i\) with respect to health. Then her expected marginal utility is always lower than the one of Agent 2: \(Eu_1^i(w - h, \tilde{H} - \tilde{\epsilon}) < Eu_2^i(w - h, \tilde{H} - \tilde{\epsilon})\).

(ii) Now, let us assume that fear of sickness, as defined by Courbage and Rey (2006), is higher for Agent 1 than for Agent 2. There exists a function \(K(.)\) satisfying \(K'(.) > 1\) and such that

\[ u^2(., \overline{H}) = u^1(., \overline{H}) \]

\[ u^2(., H) = K(u^1(., H)) \quad \forall H < \overline{H}. \]

Let us recall that \(u^i(0, 0) = 0\) for any agent \(i\). We have, for any \(h\),
\[ v^2(h) = u^2(w - h, H) - [\tilde{q}u^2(w - h, H - \epsilon) + (1 - \tilde{q})u^2(w - h, H)] \]
\[ = u^1(w - h, H) - [\tilde{q}K(u^1(w - h, H - \epsilon)) + (1 - \tilde{q})K(u^1(w - h, H))] \]
\[ = u^1(w - h, H) - [\tilde{q}u^1(w - h, H - \epsilon) + (1 - \tilde{q})u^1(w - h, H)] \]
\[ + \tilde{q}[u^1(w - h, H - \epsilon) - K(u^1(w - h, H - \epsilon)) + (1 - \tilde{q})[u^1(w - h, H) - K(u^1(w - h, H))]] \]
\[ = v^1(h) + a, \]

with \( a = \tilde{q}[u^1(w - h, H - \epsilon) - K(u^1(w - h, H - \epsilon)) + (1 - \tilde{q})[u^1(w - h, H) - K(u^1(w - h, H)))] < 0. \)

Finally, we obtain that \( V^2_h(h^1) < V^1_h(h^1) = 0 \) and \( h^2 < h^1. \) Proposition 2 is demonstrated. ◆

**Proof of Proposition 4.**

Let us consider the distribution of probability, \( G, \) such that \( F(q) - G(q) \geq 0, \forall q, \) with at least one strict inequality. The level of prevention, \( h, \) increases with \( G \) compared to \( F \) iff

\[
\int_{0}^{1} T'[U(h, q)]. \frac{\partial U(h, q)}{\partial h} dF(q) \leq \int_{0}^{1} T'[U(h, q)]. \frac{\partial U(h, q)}{\partial h} dG(q).
\]

After an integration by part, this is equivalent to

\[
\int_{0}^{1} \left( T''[U(h, q)]. \frac{\partial U(h, q)}{\partial h} \frac{\partial U(h, q)}{\partial q} + T'[U(h, q)]. \frac{\partial^2 U(h, q)}{\partial h \partial q} \right) [F(q) - G(q)] dq \geq 0 \tag{12}
\]

We have \( \frac{\partial U(h, q)}{\partial q} < 0 \) (see (1)), \( \frac{\partial^2 U(h, q)}{\partial h \partial q} > 0 \) is positive under the assumption \( u_{12} \geq 0. \)

The sign of \( \frac{\partial U(h, q)}{\partial h} \) can be positive or negative according to the values of \( q \) (see(5)).

This expression is increasing in \( q \) and negative at \( q = 0 \) if \( u_{12} \geq 0. \) Let us define \( q_1 \) as satisfying \( \frac{\partial U(h, q_1)}{\partial h} = 0. \) Consequently (12) can be rewritten as:

\[
\int_{0}^{q_1} \left( T''[U(h, q)]. \frac{\partial U(h, q)}{\partial h} \frac{\partial U(h, q)}{\partial q} + T'[U(h, q)]. \frac{\partial^2 U(h, q)}{\partial h \partial q} \right) [F(q) - G(q)] dq + \]
\[
\int_{q_1}^{1} \left( T''[U(h, q)]. \frac{\partial U(h, q)}{\partial h} \frac{\partial U(h, q)}{\partial q} + T'[U(h, q)]. \frac{\partial^2 U(h, q)}{\partial h \partial q} \right) [F(q) - G(q)] dq \geq 0 \tag{13}
\]

The second term in () is positive if \( u_{12} \geq 0. \) The first term is also positive when

\[
- \frac{T''}{T'} \leq \frac{\partial^2 U(h, q)}{\partial q \partial h} \frac{\partial U(h, q)}{\partial h} \]

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where \(-\frac{T''}{T'}\) is the coefficient of ambiguity aversion as defined by Klibanoff, Marinacci, Mukerji (2005). Proposition 4 is demonstrated. ♦

**Proof of Proposition 5.**

The first order condition of (8) writes:

\[
\int_0^1 T'[U(\alpha, q)] \frac{\partial U(\alpha, q)}{\partial \alpha} dF(q) = 0
\]  \hspace{1cm} (14)

with

\[
\frac{\partial U(\alpha, q)}{\partial \alpha} = p[qu_1(w - \Pi - (1 - \alpha)c(\epsilon), H^t). \left( c(\epsilon) - \frac{\partial \Pi}{\partial \alpha} \right) \\
+ (1 - q)u_1(w - \Pi - (1 - \alpha)c(0), H^t)]. \left( c(0) - \frac{\partial \Pi}{\partial \alpha} \right) \\
- \frac{\partial \Pi}{\partial \alpha} (1 - p)u_1(w - \Pi, \overline{H})
\]  \hspace{1cm} (15)

and

\[
\frac{\partial \Pi}{\partial \alpha} = (1 + \lambda)p \int_0^1 (qc(\epsilon) + (1 - q)c(0)) dF(q) \\
= (1 + \lambda)p \int_0^1 E_q[c(\epsilon)] dF(q) > 0.
\]  \hspace{1cm} (16)

Expression (15) at \(\alpha = 1\) writes:

\[
\frac{\partial U(\alpha, q)}{\partial \alpha} \bigg|_{\alpha = 1} = p \left[ qu_1(w - \Pi, H^t). \left( c(\epsilon) - \frac{\partial \Pi}{\partial \alpha} \right) \\
+ (1 - q)u_1(w - \Pi, H^t)]. \left( c(0) - \frac{\partial \Pi}{\partial \alpha} \right) \\
- \frac{\partial \Pi}{\partial \alpha} (1 - p)u_1(w - \Pi, \overline{H})
\] \\
= - \frac{\partial \Pi}{\partial \alpha} [pu_1(w - \Pi, H^t) + (1 - p)u_1(w - \Pi, \overline{H})] \\
+ pu_1(w - \Pi, H^t)E_q[c(\epsilon)].
\]

Expression (16) at \(\lambda = 0\) writes \(\frac{\partial \Pi}{\partial \alpha} = p \int_0^1 E_q[c(\epsilon)] dF(q)\). Expression \(U(1, q)\) is independent of \(q\). Let us rewrite it \(U_1 = U(1, q)\). The first order condition (14) becomes, for \(\alpha = 1\) and \(\lambda = 0\):
\[
\begin{align*}
&\int_0^1 T'[U_1] \cdot \frac{\partial U(1, q)}{\partial \alpha} dF(q) \\
&= -T'[U_1] \left[ \frac{\partial \Pi}{\partial \alpha} [pu_1(w - \Pi, H^t) + (1 - p)u_1(w - \Pi, \overline{H})] \\
&\quad + pu_1(w - \Pi, H^t) \int_0^1 E_q[c(\bar{\epsilon})] dF(q) \right] \\
&\quad + pu_1(w - \Pi, H^t) \int_0^1 E_q[c(\bar{\epsilon})] dF(q) \\
&= T'[U_1] p \int_0^1 E_q[c(\bar{\epsilon})] dF(q)[(1 - p)u_1(w - \Pi, H^t) - (1 - p)u_1(w - \Pi, \overline{H})] \\
&\quad + pu_1(w - \Pi, H^t) \int_0^1 E_q[c(\bar{\epsilon})] dF(q)
\end{align*}
\]

With \( H^t < \overline{H} \) by definition, this expression is positive iff \( u_{12} < 0 \) and, finally, we have \( \alpha = 1 \) in optimum. This result is obtained with \( \lambda = 0 \). However it still holds for \( 0 < \lambda \leq \lambda_0 \). Indeed, the right-hand-side term in the above expression is just modified by the added term

\[
-\lambda.p[pu_1(w - \Pi, H^t) + (1 - p)u_1(w - \Pi, \overline{H})]T'[U_1] \int_0^1 E_q[c(\bar{\epsilon})] dF(q),
\]

which is equal to zero for \( \lambda = 0 \), strictly negative for \( \lambda > 0 \) and decreasing in \( \lambda \). Since it is also continuous, there exists a strictly positive value \( \lambda_0 \) such that the first order condition for any \( \lambda < \lambda_0 \), and equal to \( 17 + 18 \) remains positive. Proposition 5 is demonstrated.

**Proof of Proposition 6.**

Let us consider two agents 1 and 2, with functions \( T_1 \) and \( T_2 \). Let us suppose that there is an increasing and concave function \( k \) such that \( T_2 = k(T_1) \). As utility function \( U(\alpha, q) \) is a decreasing function of \( q \), for all \( q \) in \( [0, 1] \), \( U(\alpha, q) \) is less than \( U(\alpha, 0) \). Then, for all \( q \) and for all increasing function \( T \), \( k'(T[U(\alpha, q)]) \) is lower than \( k'(T[U(\alpha, 0)]) \).

This implies that

\[
\int_0^1 T_2[U(\alpha^1, q)] \frac{\partial U(\alpha^1, q)}{\partial \alpha} dF(q) = \int_0^1 k'(T_1)T'_1[U(\alpha^1, q)] \frac{\partial U(\alpha^1, q)}{\partial \alpha} dF(q)
\]

\[
> k'(T_1[U(\alpha^1, 0)]) \frac{\partial U(\alpha^1, 0)}{\partial \alpha} \int_0^1 T'_1[U(\alpha^1, q)] dF(q)
\]

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when $\frac{\partial U}{\partial \alpha_{q}} > 0$. With $c(\epsilon) > c(0)$ and $u_{11} < 0$ by assumption, this is the case. Indeed, from (15) in Appendix we have

$$
\frac{\partial U}{\partial \alpha_{q}} = \rho[(c(\epsilon) - \frac{\partial H}{\partial \alpha})u_{1}(w - \Pi + I(\epsilon) - c(\epsilon), H^t) - (c(0) - \frac{\partial H}{\partial \alpha})u_{1}(w - \Pi + I(0) - c(0), H^t)].
$$

Consequently, the last term of the above inequality is positive. Finally, the first order condition of Agent 2 evaluated at $\alpha = \alpha^1$ is strictly positive, so that $\alpha^1 < \alpha^2$. Proposition 6 is demonstrated.

**Proof of Proposition 8.**

The first order conditions of program (10)-(11) are:

\[
\int_{0}^{1} T'[U(\alpha, h, q)] \frac{\partial U(\alpha, h, q)}{\partial \alpha} dF(q) = 0
\]

and

\[
\int_{0}^{1} T'[U(\alpha, h, q)] \frac{\partial U(\alpha, h, q)}{\partial h} dF(q) = 0
\]

with

\[
\frac{\partial U(\alpha, h, q)}{\partial \alpha} = p(h)[qu_{1}(w - \Pi - (1 - \alpha)c(\epsilon) - h, H^t). \left(c(\epsilon) - \frac{\partial H}{\partial \alpha}\right) + (1 - q)u_{1}(w - \Pi - (1 - \alpha)c(0) - h, H^t))] \left(c(0) - \frac{\partial H}{\partial \alpha}\right)] - \frac{\partial H}{\partial \alpha}(1 - p(h))u_{1}(w - \Pi - h, \bar{H})
\]

\[
\frac{\partial U(\alpha, h, q)}{\partial h} = p'(h)[qu_{1}(w - \Pi - (1 - \alpha)c(\epsilon) - h, H^t) + (1 - q)u(w - \Pi - (1 - \alpha)c(0) - h, H^t)] - p(h). \left(1 + \frac{\partial H}{\partial h}\right) [qu_{1}(w - \Pi - (1 - \alpha)c(\epsilon) - h, H^t) + (1 - q)u_{1}(w - \Pi - (1 - \alpha)c(0) - h, H^t)] - (1 - p(h))u_{1}(w - \Pi - h, \bar{H}) \left(1 + \frac{\partial H}{\partial h}\right)
\]

and

\[
\frac{\partial H}{\partial \alpha} = (1 + \lambda)p(h) \int_{0}^{1} (qc(\epsilon) + (1 - q)c(0)) dF(q) > 0
\]

\[
\frac{\partial H}{\partial h} = (1 + \lambda)\alpha p'(h) \int_{0}^{1} (qc(\epsilon) + (1 - q)c(0)) dF(q) < 0.
\]

The course of the proof is similar to the proof of Proposition 5. ♦
References


